CSIR MATHEMATICS
16 Feb-2022 Solution

PART "A"

Qus: (ID 34)

Sol:- (3) Given variance $(X_1) = 3^2 = 9$

& variance $(X_2) = 4^2 = 16$

$\Rightarrow \quad V(X_1 + X_2) = V(X_1) + V(X_2) = 9 + 16 = 25$

as X_1 & X_2 are independent.

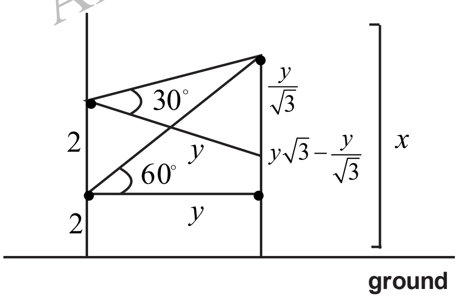
$\Rightarrow \quad$ S.D $(X_1 + X_2) = \sqrt{V(X_1 + X_2)} = \sqrt{25} = 5$

Qus:- (ID 32)

Sol:- (1) The first move can be in any direction and after that second move will be in fixed direction from available 4 directions, so required probability $= \frac{4}{4} \times \frac{1}{4} = \frac{1}{4}$

Qus:- (ID 37)

Sol:- (1)



Let x be the height of the bird from the ground.

$\Rightarrow \quad y\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 2 \quad \Rightarrow y = \sqrt{3}$

$\Rightarrow \quad x = 2 + y\sqrt{3} = 2 + \sqrt{3}\left(\sqrt{3}\right) = 5$

Qus:- (ID 30)

Sol:- (1) He $= \frac{5}{2} \times 10^{-4}$

& Kr $= 5 \times 10^{-6}$

So, $\frac{He}{Kr} = \frac{100}{2} = \frac{200}{4}$

Qus:- (ID 24)

Sol:- (2) $A + B + C = 6$ and $A, B, C \geq 1$

$\Rightarrow \quad A_1, B_1, C_1 \geq 0$ & $A = A_1 + 1, B = B_1 + 1$

& $C = C_1 + 1$ so that

$A_1 + B_1 + C_1 = 3$ whose solutions are

$3 + 3 - 1_{C_{3-1}} = 5_{C_2} = 10$

Qus:- (ID 23)

Sol:- (3) Let ratio of A be x and ratio of B be y then

$4x + 6x + 30x + 20x = 3y + 10y + 35y + 30y + 6$

$\Rightarrow \quad 60x = 78y + 6 \quad \Rightarrow \quad 10x = 13y + 1$

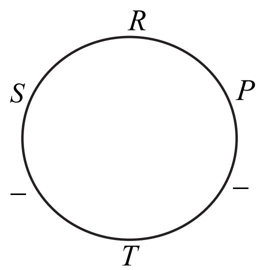
$\Rightarrow \quad y = 3$ & $x = 4$

So, number of coins with $A = 4(15) = 60$

& number of coins with $B = 3(18) = 54$

Qus:- (ID 28)

Sol:- (3)



S & T cannot seat diametrically opposite to each other.

Qus:- (ID 22)

Sol:- (4) Let xy be the two digit number then

$$(10x+y)^2 = 100c + 10x + y$$

$$\Rightarrow \quad (10x+y)(10x+y-1) = 100c$$

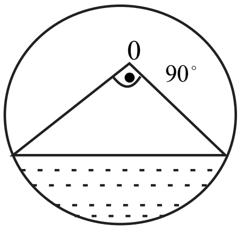
At $C=6$ it will become

$$(10x+y)(10x+y-1) = 25\times 24$$

$$\Rightarrow \quad x=2;\ y=5\ \&\ c=6$$

Qus:- (ID 27)

Sol:- (2)



Area of shaded portion =

$$\frac{1}{4}\pi(1)^2 - \frac{1}{2}\times 1\times 1 =$$

$$\frac{\pi}{4}-\frac{1}{2} = \frac{1}{4}(\pi-2)$$

Qus:- (ID 21)

Sol:- (4) Number of students in section B is x then

$$x\times 1 = 24\times(1.25)$$

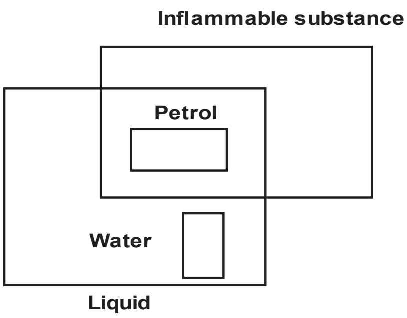
$$\Rightarrow \quad x=30$$

Qus: (ID 36)

Sol:- (2) It is impossible to get number of grains equal in each measurement.

Qus: (ID 33)

Sol:- (1)



Venn diagram representation.

Qus:- (ID 29)

Sol:- (1) 01012000 to 31122020

palindrome will be of the form

fixed

0 2 2 0

— — — — — — — —

0 1	1 0
0 2	2 0
1 0	0 1
1 1	1 1
2 0	0 2
2 1	1 2

So there are 6 palindromes

01022010,

02022020,

10022001, 11022011,

20022002 & 21022012

Qus:- (ID 38)

Sol:- (1) Density of 1,2,3 & 4 km radius are respectively $\frac{6}{\pi}, \frac{13}{4\pi}, \frac{28}{9\pi}\ \&\ \frac{50}{16\pi}$

So, density is maximum within 1 km radius.

Qus:- (ID 40)

Sol:- (1) Graph in option 1 correctly represents change in index.

Qus:- (ID 26)

Sol:- (3) The given graph shows that cell abundance was limited by the availability of nutrients but a high nutrient concentration seems toxic for the bacteria.

Qus:- (ID 25)

Sol:- (2) $(40)(x) = (40+10)(x-2)$

$$\Rightarrow \quad 4x = 5x-10 \quad \Rightarrow x=10$$

Qus:- (ID 39)

Sol:- (3) Let h & R be height and radius of larger jar then

Volume of larger jar

$V_L = \pi R^2 h$

and volume of smaller jar

$V_S = \pi \left(\frac{R}{10}\right)^2 h$

so, number of smaller jars to be filled is $\frac{V_L}{V_S} = \frac{\pi R^2 h}{\pi \left(\frac{R}{10}\right)^2 h} = 100$

Qus:- (ID 35)

Sol:- (2) If all planets move in the same plane then the visible planets will appear alinged along a straight line in the sky as seen from the earth.

Qus:- (ID 31)

Sol:- (2) After rotating by 90° about centre we get option (2)

"Part B":-

Qus:- (ID 476)

Sol:- (4) Sum of the elements in each column of BA^T will be $2\times 2 = 4$, so 4 is an eigenvalue of BA^T. Hence an eigen value of $I - \frac{1}{4}BA^T$ will be $1-\frac{1}{4}\times 4 = 1-1 = 0$

Hence $I - \frac{1}{4}BA^T$ will be singular.

Qus:- (ID 478)

Sol:- (3)

$M = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 3 \end{bmatrix}$ has 1 eigenvalue 1 and

let x & y be two other eigenvalues of M then

$1 + x + y =$ trace $(M) = 5$

& $1.x.y = \det(M) = 4$

$\Rightarrow \quad x + y = 4$ & $xy = 4 \Rightarrow x = y = 2$

So, eigen values of M are $1, 2, 2$.

Geometric multiplicity of eigen value 2 of $M =$

$3 -$ Rank $(M - 2I) =$

$$3 - \text{Rank}\left(\begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}\right) =$$

$3 - 2 = 1$

So, G.M of eigenvalue 2 = 1

Also G.M of eigenvalue 1 = 1

Hence the eigenspace of each eigenvalue of M has dimension 1.

Qus:- (ID 474)

Sol:- (4)

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots \quad (1)$$

$$\log_e(1-x) = -\left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\ldots\right] \quad \ldots (2)$$

By putting $x = \frac{1}{3}$ in (1) we get

$$S_1 = \log_e\left(1 + \frac{1}{3}\right) = \log_e\left(4/3\right)$$

By putting $x = \frac{1}{4}$ in (2) we get

$$-S_2 = \log_e\left(1 - \frac{1}{4}\right) = \log_e\left(3/4\right)$$

$\Rightarrow \quad S_2 = \log_e\left(4/3\right)$

$\Rightarrow \quad S_1 = S_2$

Qus:- (ID 473)

Sol:- (3) $f(x) = x^2$ & $g(x) = \sin x$

$\Rightarrow \quad h(x) = g(f(x)) = \sin x^2$

Which is not uniformly continuous in R

$h(x) = f(g(x)) = \sin^2 x$ is continuous periodic function in R so it is uniformly continuous in R.

$h(x)=x^2\sin x=f(x)g(x)$ &

$h(x)=x^2+\sin x=f(x)+g(x)$

are not uniformly continuous in R.

Qus:- (ID 472)

Sol:- (1) The set of all polynomials with rational coefficients is countable as it is equivalent to $\underbrace{Q\times Q\times\ldots\times Q}_{n\ times}$ which is countable.

Qus:- (ID 475)

Sol:- (1) $Null(N)=\{X \mid NX=0\}$

$$=\left\{(x_1,x_2,x_3,x_4)\middle| \begin{array}{l} x_1+x_2+x_3-x_4=0 \\ \&\ x_1-x_2+x_3+x_4=0\end{array}\right\}$$

So, $(1,2,1,4)\notin Null(N)$

hence statement I is false

For statement II:-

If $\left\{(1,1,1,0)^T,(1,0,1,1)^T\right\}$ is a basis of $col(M)$ then vectors of $col(M)$ are of the form

$a(1,1,1,0)+b(1,0,1,1)=$

$(a+b,a,a+b,b)$

Hence $(1,1,1,1)^T$ & $(1,0,1,0)^T$ do not belongs to $col(M)$ so they belongs to $Null(M)$ hence statement II is true.

Qus:- (ID 469)

Sol:- (3) In subsets of S whose intersection with subsets A and B are non-empty from 80 elements outside A & B choose any number of elements from then and choose at least one element of A and atleast one element of B. So, number of such subsets will be

$$\left(80_{C_0}+80_{C_1}+\ldots+80_{C_{80}}\right)\left(10_{C_1}+10_{C_2}+\ldots+10_{C_{10}}\right)^2$$

$$=\left(2^{80}\right)\left(2^{10}-1\right)^2$$

Qus:- (ID 470)

Sol:- (2) $\lim\limits_{n\to\infty}\frac{1}{n}\left(1+\sqrt{2}+\sqrt[3]{3}+\ldots.+\sqrt[n]{n}\right)$

$$=\lim_{n\to\infty}\frac{1}{n}\left(1+2^{1/2}+3^{1/3}+\ldots.+n^{1/n}\right)$$

$$=\lim_{n\to\infty}n^{1/n}$$

$=1$ (By Cauchy's first theorem on limit)

Qus:- (ID 480)

Sol:- (1)

$V=\left\{A\in M_{3\times3}(R):\ A^t+A\in R.I\right\}$

So, V is collection of those matrices whose sum with it's transpose is scalar matrix.

$$\Rightarrow \quad V=\left\{\begin{bmatrix} a & d & c \\ -d & a & b \\ -c & -b & a\end{bmatrix}\middle| a,b,c,d\in R\right\}$$

Now $q(A)=(\text{Trace}(A))^2-\left(\text{Trace}\left(A^2\right)\right)$

$$\Rightarrow q(a,b,c,d)=(3a)^2-\left(3a^2-\left(2b^2+2c^2+2d^2\right)\right)$$

$$=6a^2+2b^2+2c^2+2d^2$$

which is positive definite so it's signature is $(+,+,+,+)$

Qus:- (ID 479)

Sol:- (2) Inner product (Standard) on $M_n(R)$ is $\langle A,B\rangle=$ trace $\left(AB^T\right)$ and for symmetric matrices it will reduce to

$\langle A,B\rangle=\text{trace}(AB)$

Qus:- (ID 477)

Sol:- (3) Eigen values of A are $-1,1,1,-2$ &

$B=A^4-5A^2+5I=\left(A^2-I\right)\left(A^2-4I\right)+I$

$\Rightarrow$ eigen values of B are $1,1,1\&1$ so eigen values of $A+B$ are $0,2,2,-1$ so

trace $(A+B)=0+2+2+(-1)=3$

Qus:- (ID 471)

Sol:- (2)

$$a_n=3+5\left(-\frac{1}{2}\right)^n+(-1)^n\left(\frac{1}{4}+(-1)^n\frac{2}{n}\right)$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \left\{3-\frac{1}{4}, 3+\frac{1}{4}\right\} = \left\{\frac{11}{4}, \frac{13}{4}\right\}$$

$$\Rightarrow \quad \liminf_{n\to\infty} a_n = \frac{11}{4} \ \& \ \limsup_{n\to\infty} a_n = \frac{13}{4}$$

$$\Rightarrow \quad \left(\liminf_{n\to\infty} a_n, \limsup_{n\to\infty} a_n\right) = \left(\frac{11}{4}, \frac{13}{4}\right)$$

Qus:- (ID 486)
Sol:- (2) Number of generators of a cyclic group of order 36 =

$$\phi(36) = \phi(2^2\times 3^2) = (2^2-2)(3^2-3)$$

$$= 2\times 6 = 12$$

Qus:- (ID 483)
Sol:- (2)

$$f(z) = \frac{z^2+2z-4}{z}$$

fails to be analytic at $z=0$ so radius of convergence of $f(z)$ about $z=1$ will be $R = |1-0| = 1$

Qus:- (ID 482)
Sol:- (2)

$f\left(\frac{1}{n}\right) = e^{-n} \Rightarrow f(z) = e^{-\frac{1}{z}}$ which is not analytic in $|z|<1$, so A is empty set.

$$B = \left\{f \mid f\left(\frac{1}{n}\right) = \frac{(n-2)}{(n-1)}\right\}$$

$$= \left\{f \mid f\left(\frac{1}{n}\right) = \frac{\left(1-\frac{2}{n}\right)}{\left(1-\frac{1}{n}\right)}\right\}$$

$$= \left\{f \mid f(z) = \frac{1-2z}{1-z}\right\}$$

which is analytic in $|z|<1$

So, $B = \left\{f(z) = \dfrac{1-2z}{1-z}\right\}$ which is singleton set.

Qus:- (ID 487)
Sol:- (2) R can have exactly two maximal ideals R is commtative ring.
Recall the fact that
If R is C.R.U then R has at least one maximal ideal.
and also in C.R.U every maximal ideal is prime.
Hence correct option is 2
i.e. R can have exactly two maximal ideal.

Qus:- (ID 484)
Sol:- (4)

$$I = \int_\gamma \frac{e^{i\pi z}}{(z-1)(z-2i)^2}\,dz \ ; \ \gamma : |z| = \frac{3}{2}$$

inside γ integrand $f(z)$ has only singularity as simple pole at 1. Hence

$$I = 2\pi i \operatorname{Res}_{z=1} f(z) = 2\pi i \frac{e^{i\pi}}{(1-2i)^2}$$

So, $I = 2\pi i c$

$$\Rightarrow \quad c = \frac{e^{i\pi}}{(1-2i)^2}$$

$$\Rightarrow \quad |c| = \left|\frac{e^{i\pi}}{(1-2i)^2}\right| = \frac{1}{(\sqrt{5})^2} = \frac{1}{5}$$

Qus:- (ID 488)
Sol:- (1) $d(f(x), f(y)) \le d(x,y) \ ; \ \forall x, y \in X$

$\Rightarrow$ $\forall \in > 0, \ \exists \delta = \in$ for any $c \in X$ such that
$d(f(x), f(c)) < \in$ whenever
$d(x,c) < \delta$
so, $f(x)$ is continuous function.

Qus:- (ID 481)
Sol:- (2) $u_x = v_y$ is correct and $u_x = -v_y$ is false.

Qus:- (ID 485)
Sol:- (2) $S = \{n : 1 \le n \le 999 \ ; \ 3 \mid n \text{ or } 37 \mid n\}$

$$\Rightarrow \quad |S| = \left[\frac{999}{3}\right] + \left[\frac{999}{37}\right] - \left[\frac{999}{111}\right]$$

$$= 333 + 27 - 9 = 351$$

$\Rightarrow \quad |S^c| = 999 - |S| = 999 - 351$

$= 648$

Qus:- (ID 494)

Sol:- (3) $I(y) = \int_{-1}^{1} (y'^2 - 2xy)\, dx$

By euler equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad \Rightarrow -2x - \frac{d}{dx}(2y') = 0$$

$\Rightarrow \quad -2x - 2y'' = 0$

$y'' + x = 0 \qquad \Rightarrow y'' = x$

$$y' = \frac{-x^2}{2} + c_1$$

$$y = \frac{-x^3}{6} + c_1 x + c_2$$

Using $y(-1) = -1$

& $y(1) = 1$

$\Rightarrow \boxed{c_1 = \frac{7}{6}}$ & $\boxed{c_2 = 0}$ $\Rightarrow y(x) = \frac{-x^3}{6} + \frac{7}{6}x$

Qus:- (ID 491)

Sol:- (1) $\frac{\partial u'}{\partial y} - x\frac{\partial u'}{\partial x} + u - 1 = 0 \quad u(x,0) = \sin x$

Lagrange's eq. is $\frac{dx}{-x} = \frac{dy}{1} = \frac{du'}{1-u}$

From (i) and (ii) we get

$\log x = -y + \log c_1$

$\Rightarrow \boxed{c_1 = x e^y}$

From (i) and (ii) we get

$-\log x = -\log(1-u) + \log c_2$

$\Rightarrow \boxed{\frac{1-u}{x} = c_2}$

$\phi(c_1) = c_2$

$$\phi(x e^y) = \frac{1-u}{x} \Rightarrow \phi(x) = \frac{1-\sin x}{x}$$

$$\Rightarrow \quad \frac{1-u}{\cancel{x}} = \frac{1-\sin(x e^y)}{\cancel{x}\, e^y}$$

$$\Rightarrow \quad 1 - u = \frac{1-\sin(x e^y)}{e^y}$$

$$u = 1 - \frac{1-\sin x e^y}{e^y}$$

$$u(0,1) = 1 - \frac{1-0}{e}$$

$$u(0,1) = \boxed{1 - \frac{1}{e}}$$

Qus:- (ID 496)
Sol:- (2)

Qus:- (ID 495)
Sol:- (1)

$\int_{-1}^{1} f(x)\, dx = a f(-1) + b f'(0) + c f'(1)$ (1)

is exact for polynomials of degree upto 2 if it is exact for $f(x) = 1$, $f(x) = x$ & $f(x) = x^2$

(i) $f(x) = 1 \Rightarrow f'(x) = 0$ so (1) becomes

$\int_{-1}^{1} 1\, dx = a(1) + b(0) + c(0)$

$\Rightarrow \quad a = 2$ (2)

(ii) $f(x) = x \quad \Rightarrow f'(x) = 1$ so, (1) becomes

$\int_{-1}^{1} x\, dx = 2(-1) + b(1) + c(1) \Rightarrow b + c - 2 = 0$

$\Rightarrow \quad b + c = 2$

Hence $a + b + c = 2 + 2 = 4$

Qus:- (ID 489)

Sol:- (2) $4x y'' + 2y' + y = 0$ (i)

Let $y = A + Bx + Cx^2$

$y' = B + 2Cx,\ y'' = 2C$

Putting in (i)

$4x(2C) + 2x(B + 2Cx) + A + Bx + Cx^2 = 0$ (*)

Using $y(0) = 1 \qquad \Rightarrow \boxed{A = 1}$

Put in (*)

$8Cx + 2(B + 2Cx) + 1 + Bx + Cx^2 = 0$

$Cx^2 + (12C + B)x + (2B + 1) = 0$

$$\boxed{B = \frac{-1}{2}}$$

$$12C - \frac{1}{2} = 0 \Rightarrow \boxed{C = \frac{1}{24}}$$

$$\Rightarrow\; y'' = 2C \Rightarrow 2 \times \frac{1}{24} = \frac{1}{2}$$

Qus:- (ID 493)
Sol:- (3) I.V.P is

$$\left.\begin{aligned} y' &= y - t^2 + 1 \\ y(0) &= 0.5 \\ 0 \le t &\le 2 \end{aligned}\right\} \quad (1)$$

$$\frac{dy}{dt} - y = -t^2 + 1 \Rightarrow (D-1)y = -t^2 + 1$$

So, complementary function is $y = ce^t$ and particular integral is

$$y = \frac{1}{D-1}(-t^2+1) = \frac{1}{(D-1)}(t^2-1)$$

$$= (1 + D + D^2 + ...)(t^2 - 1)$$

$$= t^2 - 1 + 2t + 2 = (t+1)^2$$

Hence $y = ce^t + (t+1)^2$

$$y(0) = 0.5 \Rightarrow 0.5 = c + 1 \Rightarrow c = -0.5$$

$$\Rightarrow\; y = -(0.5)e^t + (t+1)^2 \quad (2)$$

Actual solution

Hence $y(0.8) = -(0.5)e^{0.8} + (1.8)^2$

By Euler's method

$$f(t,y) = y - t^2 + 1$$

$$\Rightarrow\; f(0, 0.5) = 1.5$$

So, $W(0.4) = 0.5 + (0.4)(1.5) = 1.1$

$$f(0.4, 1.1) = 1.1 - (0.4)^2 + 1 = 1.94$$

$$w = (0.8) = 1.1 + (0.4)(1.94) = 1.876$$

So, error in approximation

$$E = y(0.8) - w(0.8)$$

$$= -(0.5)e^{0.8} + (1.8)^2 - 1.876$$

Error bound by Euler's method is

$$|w_i - y_i| \le \frac{Mh}{2L}\left(e^{L(t_i - a)} - 1\right) \text{ in interval } [a,b]$$

L is Lipschitz constant which is 1

i.e. $L = 1$; $h = 0.4$, $M = \sup_{t\in[a,b]} |y''(t)|$

$$\Rightarrow\; M = \sup_{t\in[0,2]} \left|-0.5e^t + 2\right| = (0.5)e^2 - 2$$

$a = 0$ & $t_i = 0.8$

So, $|w_i - y_i| \le \dfrac{\left[(0.5)e^2 - 2\right](0.4)}{2}\left(e^{1(0.8-0)} - 1\right)$

$$\Rightarrow\; |w_i - y_i| \le 0.2\left((0.5)e^2 - 2\right)\left(e^{0.8} - 1\right)$$

So, $(0.2)\left((0.5)e^2 - 2\right)\left(e^{0.8} - 1\right)$

Qus:- (ID 490)
Sol:- (2)

$$\frac{dx}{dt} = x^3,\; x(0) = 1 \quad \text{(i)}$$

$$\frac{dx}{dt} = x\sin x^2,\; x(0) = 2 \quad \text{(ii)}$$

By solving (i) we get

$$x^2 = \frac{1}{1-2t} \quad \text{(A)}$$

$$\Rightarrow\; x = \pm\frac{1}{\sqrt{1-2t}} \Rightarrow 1 - 2t = 0 \Rightarrow t = \frac{1}{2}$$

$\Rightarrow$ Solution (A) blows up in finite time

Also, $x'(t) = x\sin x^2$

$$\Rightarrow\; x(t) = -\cos\frac{x^2}{a} + c_2$$

$$x(0) = 1$$

$$\Rightarrow\; x(t) = -\cos\frac{x^2}{2} + \frac{5}{2} \Rightarrow x(t) \text{ bdd in } \mathbb{R} \text{ .(b)}$$

$\Rightarrow$ Solution (b) not blows up in finite time.

Qus:- (ID 492)

Sol:- (1) Using $S^2 - 4RT = 0$ if given PDE is parabolic we get (1) option as not parabolic.

Qus:- (ID 503)
Sol:- (4)

Qus:- (ID 499)
Sol:- (2)

Qus:- (ID 508)
Sol:- (4)

Qus:- (ID 501)
Sol:- (3)

Qus:- (ID 505)
Sol:- (3)

Qus:- (ID 504)
Sol:- (3)

Qus:- (ID 506)
Sol:- (2)

Qus:- (ID 498)
Sol:- (1)

Qus:- (ID 497)
Sol:- (3)

Qus:- (ID 500)
Sol:- (4)

Qus:- (ID 502)
Sol:- (1)

Qus:- (ID 507)
Sol:- (2)

Part "C"

Qus:- (ID 523)

Sol:- (4) Let $X_0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Further take $A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

Now $AX_0 - X_0 A = A \Rightarrow$

$$\begin{pmatrix} a+c & b+d \\ -a-c & -b-d \end{pmatrix} - \begin{pmatrix} a-b & a-b \\ c-d & c-d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$\Rightarrow \quad b+c=1,\ 2b+d-a=1,\ -a-2c+d=-1$

$-b-c-=-1$

$\Rightarrow \quad b+c=1$ & $a+2c=d+1$

$|X_0| = ad - bc = a(a+2c-1) - c(1-c)$

$= a^2 + 2ac - a + c^2 - c$

$= (a+c)^2 - (a+c)$

$= (a+c)(a+c-1)$

Which is product of two consecutive integers. So, $0, 2, 6$ are possible but 10 is not possible as it is not product of two consecutive integers.

Qus:- (ID 520)
Sol:- (2,3,4)

I:- If $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ then for no non-zero vectors $b_2, b_3 \in R^m$.

$AX = b_2$ has a solution and also

$AX = b_3$ has no solution, so option (1) is false.

II:- Whenever A is singular then $\exists b_1 \neq 0$ such that $AX = b_1$ has no solution, so option 2 is correct.

III:- If $m = 3$ i.e. A is 3×3 matrix of rank 2 then both I and II will become simultaneously true.

IV:- If $m = 2$ and I is true then Rank $(A) = 0$ or 1 and in this case we cannot have 2 linearly independent vectors b_2 and b_3 such that $AX = b_2$ & $AX = b_3$ have solutions hence II will be false.
Hence option 4 is correct.

Qus:- (ID 517)

Sol:- (3,4) Derivative map of $g(x, y, z)$ at $(0,0,0)$ is $(1,1,1) = B$ say then the derivative map of

$h=(f,g)$ is $\begin{pmatrix}A\\B\end{pmatrix}$ and h will admit differentiable inverse if Rank $\begin{pmatrix}A\\B\end{pmatrix}=3$ or $\det\begin{pmatrix}A\\B\end{pmatrix}\neq 0$.

In option 1, Rank $\begin{pmatrix}A\\B\end{pmatrix}=2\neq 3$

In option 2, Rank $\begin{pmatrix}A\\B\end{pmatrix}=2\neq 3$

In option 3, Rank $\begin{pmatrix}A\\B\end{pmatrix}=3$

In option 4, Rank $\begin{pmatrix}A\\B\end{pmatrix}=3$

So option 3 & 4 are correct.

Qus:- (ID 511)

Sol:- (3,4) $f:R\to R$ is continuous function

(1) If $A=R$ and $f(x)=\tan^{-1}x$ then $f(A)=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. So A is closed set but $f(A)$ is not closed set.
Hence option (1) is false

(2) If $A=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $f(x)=\tan^{-1}x$ then $f^{-1}(A)=R$. Here A is bounded but $f^{-1}(A)$ is not bounded.

(3) If A is closed and bounded i.e. A is compact then $f(A)$ is compact i.e. closed and bounded.

(4) $f:R\to R$ is continuous and A is bounded so $f(A)$ is bounded as for every $c\in R$ $\lim\limits_{x\to c} f(x)$ exist.

Qus:- (ID 515)

Sol:- (1,3) $f_K(X)=\frac{x^K}{(1+x)^2}$; $\forall x\geq 0$ &

$K\geq 1, K\in I$ (1)

For each K, $f_K^{'}(x)$ is bounded on compact intervals so f_K has bounded derivative in every compact interval so by bounded derivative theorem f_K is function of bounded variation on compact intervals
(option 1)

$\forall K\geq 1$, $\int_0^\infty \frac{x^K}{(1+x)^2}dx=\infty$

So, option 2 is false

$$f_K(x)=\frac{x^K}{(1+x)^2}=\frac{a}{(1+x)}+\frac{b}{(1+x)^2}+$$
$$c_1x^{K-2}+c_2x^{K-3}+....c_{K-1}$$

for some $a,b,c_1,c_2,....,c_{K-1}\in R$

So, $\int_0^1 f_K(x)dx=a\ln(1+x)-\frac{b}{(1+x)}+$

$$c_1\frac{x^{K-1}}{K-1}+...+c_{K-1}x\ \Big|_0^1$$
$$=a\ln 2+\frac{1}{2}b+\frac{c_1}{K-1}\quad ,+\frac{c_2}{K+2}+...+c_{K-1}$$

$$\Rightarrow\quad \lim_{K\to\infty}\int_0^1 f_K(x)dx=a\ln 2+\frac{1}{2}b+c_{K-1}$$

So, it exist $\forall K\geq 1$ (option 3)

$\{f_K(x)\}=\left\{\frac{x^K}{(1+x)^2}\right\}$ is sequence of continuous function in $[0,1]$ and pointwise limit function,

$$f(x)=\lim_{K\to\infty} f_K(x)=\lim_{K\to\infty}\frac{x^K}{(1+x)^2}$$

$=\ 0$ if $x\in[0,1)$

$=\frac{1}{4}$ if $x=1$

So, $f(x)$ is discontinuous function in $[0,1]$ hence convergence of $\{f_n(x)\}$ to $f(x)$ is pointwise but not uniform.
So option (4) is false.

Qus:- (ID 521)
Sol:- (2,3)

$M \in M_n(R)$ and $M^2 = 0$

$\Rightarrow$ Rank $(M) \le \frac{n}{2}$ & Nullity $(M) \ge \frac{n}{2}$

(By Rank $(AB) \ge$ Rank $(A)+$ Rank $(B)-n$ and putting $B = A$)
Hence number of linearly independent columns of $M \le$ number of linearly dependent columns of M.

$\Rightarrow$ $\dim(Col(M)) \le \dim(Null(M))$
Hence if n is odd then
$\dim(Col(M)) < \dim(Null(M))$
& if n is even then
$\dim(Col(M)) \le \dim(Null(M))$

Qus:- (ID 516)
Sol:- (1,3)

$\because$ $g_t(y) = f(t,y)$ & $h_t(x) = f(x,t)$ are non decreasing functions i.e. f is non-decreasing function w.r.t. both tuples hence $f(x,x)$ is also nondecreasing function.
As $f(x,y)$ is nondecreasing and bounded so $\lim\limits_{(x,y)\to(+\infty,+\infty)} f(x,y)$ exist.

Qus:- (ID 525)

Sol:- (1) E is connected $\Rightarrow \overline{E}$ is connected

$E = (0,1)$ & $X = R \Rightarrow \partial E = \{0,1\}$
Here E is connected but ∂E is not connected.

If $X = R^2$ and $E = \left\{\left(x, \sin\frac{1}{x}\right); x > 0\right\}$ is path connected but $\overline{E}$ is not path connected.

Qus:- (ID 509)

Sol:- (1) If $\{a_n\}$ & $\{b_n\}$ are sequences then
$\limsup\limits_{n\to\infty}(a_n + b_n) \le \limsup\limits_{n\to\infty} a_n + \limsup\limits_{n\to\infty} a_n$
(option 1)
but $\liminf\limits_{n\to\infty}(a_n + b_n) \ge \liminf\limits_{n\to\infty} a_n + \liminf\limits_{n\to\infty} b_n$
so option 3 is incorrect.

If $E = \left\{100, 1, \frac{1}{2}, \ldots \frac{1}{n}, \ldots\right\}$ &

$F = \left\{1000, 1, \frac{1}{2}, \ldots, \frac{1}{n}, \ldots.\right\}$

then $\limsup(E) = 0$
$\limsup(F) = 0$
& $\limsup(E+F) = 1000$
so, 2 & 4 are incorrect.

Qus:- (ID 524)
Sol:- (2,4)

I:- $\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2$

has matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $|A| = -3$, so A is not positive definite hence it is not positive definite.

II:- $\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$

has matrix, $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $|A| = 1 > 0$ so, it is positive definite as it's first order principal minor is also positive, hence it is positive definite.

III:- $\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$

has it's matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $|A| = 0$ so it is not positive definite.

IV:- $\left(\begin{pmatrix}x_1\\x_2\end{pmatrix},\begin{pmatrix}y_1\\y_2\end{pmatrix}\right)=x_1y_1-\frac{1}{2}x_1y_2-\frac{1}{2}x_2y_1+x_2y_2$

has matrix $A=\begin{pmatrix}1 & -1/2\\ -1/2 & 1\end{pmatrix}$

has it's minors as 1 and $\frac{3}{4}$ so it is positive definite hence it also represents inner product.

Qus:- (ID 519)

Sol:- (1,3) As A has r linearly independent rows so Rank$(A)\ge r$.

Also A has S linearly independent columns so Rank$(A)\ge S$

Hence, Rank $(A)\ge \max(r,S)$ (1)

If $r<S$ then as Rank $(A)\ge S$

So, there exist a row among $r+1,...,m$ such that together with first r rows it is linearly independent set.

Qus:-(ID 518)

Sol:- (4) $T:X\to Y$ is bounded linear operator from X to Y then T has a bounded inverse if $\inf\limits_{\|X\|=1}\|Tx\|>0$ and $T(x)$ is dense in Y.

Qus:- (ID 512)

Sol:- (1,3) $f:(0,1)\to(0,1]$ given by

$$f(x)=2x \;;\; 0<x\le\frac{1}{2}$$
$$=1 \;;\; \frac{1}{2}<x<1$$

is continuous onto function
(option 1)
Under continuous function a closed interval is mapped onto a closed interval.

$f:(0,1)\to R$ given by

$f(x)=\tan\left(x-\frac{1}{2}\right)\pi$ is continuous onto function
(option 3)
Under continuous function an interval can be mapped onto an interval only.

Qus:- (ID 522)

Sol:- (1,4)

$A^K=I \Rightarrow A^K-I=0$, So an Annihilating polynomial of A is

$(x^K-1)=(x-1)(x-w)...(x-w^K)$

So, A will be diagonalisable
(Option 1)

$A=\begin{pmatrix}0&1\\0&0\end{pmatrix}$ is not diagonalisable

but $A^2=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ is diagonalisable

Also A is nilpotent but not diagonalisble.

Also, If A has n linearly independent eigenvector then A is diagonalisable.

Qus:- (ID 526)

Sol:- (1,3) Given system is

$$\left[\begin{array}{ccc|c}2&K&0&2-K\\K&2&0&K\\0&K&K&K-1\end{array}\right]\sim$$

$$\left[\begin{array}{ccc|c}2&K&0&2-K\\K-2&2-K&0&2(K-1)\\0&K&K&K-1\end{array}\right] \quad (1)$$

Determinant of coefficient matrix

$$=\begin{vmatrix}2&K&0\\K&2&0\\0&K&K\end{vmatrix}=K(4-K^2)=0$$

If $K=0,2$ or -2

If $K\ne 0,2$ or -2 then system has unique solution

For $K=0$, system (1) is

$\begin{bmatrix}2&0&0&2\\-2&2&0&-2\\0&0&0&-1\end{bmatrix}$ so it has no solution as

Rank $(A) \neq$ Rank $(A:B)$

For $K=2$, system (1) is

$$\begin{bmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$$

So in this case also it has no solution as Rank$(A) \neq$ Rank $(A:B)$

For $K=-2$, system (1) is

$$\begin{bmatrix} 2 & -2 & 0 & 4 \\ -4 & 4 & 0 & -6 \\ 0 & -2 & -2 & -3 \end{bmatrix}$$

which has no solution as Rank $(A) \neq$ Rank$(A:B)$

Hence system is consistent for all K except $0, 2 \& -2$.

Qus:- (ID 514)

Sol:- (1,2) $\int_0^t f(x)\,dx = \int_t^1 f(x)\,dx \;;\; \forall t \in [0,1]$

Differentiating both sides of (1) w.r.to 't' we get

$f(t) = -f(t) \;;\; \forall t \in [0,1]$

$\Rightarrow \quad 2f(t) = 0 \;;\; \forall t \in [0,1]$

$\Rightarrow \quad f(t) = 0 \;;\; \forall t \in [0,1]$

Hence $f(x) = 0 \;;\; \forall x \in [0,1]$

So, $f(x)$ is differentiable in $(0,1)$ and also it is monotonic in $[0,1]$ option 1 & option 2 are correct.

Also $\int_0^1 f(x)\,dx = 0$ & $f(x) > 0$ for all rationals.

Qus:- (ID 510)

Sol:- (2,4)

$g(x) = e^x f(x) \Rightarrow f(x) = e^{-x} g(x)$

$\Rightarrow \quad f'(x) = e^{-x}\left(g'(x) - g(x)\right)$

$f(x) + f'(x) = e^{-x} g'(x)$

If $a < b$ and $b = a+h$ then by Cauchy's mean value theorem

$$\frac{g(b)-g(a)}{e^b - e^a} = \frac{g'(c)}{e^c} \;;\; c \in (a,b)$$

$$\Rightarrow \quad \lim_{\substack{a\to\infty \\ b\to\infty}} \frac{g(b)-g(a)}{e^b - e^a} = \lim_{c\to\infty} \frac{g'(c)}{e^c} = \lim_{c\to\infty} e^{-c} g'(c)$$

So, if $\lim_{x\to\infty} f(x) + f'(x) = 0$ then

$$\lim_{\substack{x\to\infty \\ y\to\infty}} \frac{g(x)-g(y)}{e^x - e^y} = 0$$

(Option 2)

Also, $\lim_{x\to\infty}\left(f(x) + f'(x)\right) = 0$

$$\Rightarrow \quad \lim_{x\to\infty} \frac{g'(x)}{e^x} = 0$$

But $\lim_{x\to\infty} \frac{g(x)}{e^x} = \lim_{x\to\infty} \frac{g'(x)}{e^x}$

(By L' hospital rule), so $\lim_{x\to\infty} \frac{g(x)}{e^x} = 0$

$$\Rightarrow \quad \lim_{x\to\infty} f(x) = 0$$

Qus:- (ID 513)

Sol:- (1,2)

$\overline{Y} \subseteq \bigcup_{j\geq 1} U_j$ and $\overline{Y}$ is compact and $\bigcup_{j\geq 1} U_j$ is an open cover of $\overline{Y}$ so this open cover admits finite subcover hence

$\overline{Y} \subseteq \bigcup_{K=1}^{N} U_{jK}$ for some $n \in$ natural number

hence option (1) & (2) are correct.

Qus:- (ID 535)

Sol:- (2,3)

Option (a) is incorrect

$Q\left(2^{1/3}\right) \mid Q$ is not Galois extension over Q as it is not normal extension.

Option (b) is correct.

Consider the extension field over Q through the polynomial $p(x) = x^3 - 3x - 1$ and $p(x)$ is normal polynomial over Q.

Hence the extension field over Q is normal . (Clearly it is finite and separable.)
Hence it is Galois extension.

Option (c) correct.
Finit extension of a finite field is Galois extension, hence correct.

Option (d) incorrect.
Degree 2 extension of a field of char. zero is Galois extension.
hence correct option (b), (c)

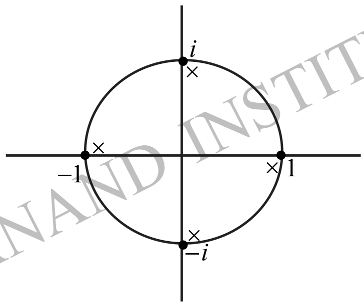
Qus:- (ID 528)
Sol:- (2,3)

$$f(z)=\sum_{n=0}^{\infty} z^{4n} = \sum_{n=0}^{\infty}\left(z^4\right)^n =$$

$$1+z^4+z^8+.....=\frac{1}{1-z^4}$$

$\Rightarrow$ $f(z)$ has 4 singular point $1,-1,i\,\&-i$ because they are 4 solutions of $1-z^4=0$

Hence $R=T\setminus\{1,-1,i,-i\}$



Hence R contains all but finite number of points of T i.e. it contains infinite points of T.

Qus:- (ID 534)
Sol:- (2,4)
Option (a) incorrect.

Take S_4, S_4 has not normal subgroup of order 3.

Option (b) correct.
Group of order 24 is not simple group

$O(G)=24=2^3\times 3$

as $2^3|24$ but $2^4\times 24$ then subgroup of 8 is 2 S.S.G in G

$\Rightarrow$ $n_2=1+2K$; $K=0,1,2,...$

and $n_2|24$

$\Rightarrow$ $n_2=1$ or 3 (possible) (1)

Simally

$3|24$ but 3^2 does not exist 24 then subgroup of order 3 is 3.S.S.G

$n_3=1+3K$; $K=0,1,2,...$ with $n_3|24$

$\Rightarrow n_3=1$ or 4 (possible) (2)

From (1) & (2) of $n_2=1$ or $n_3=1$ implies G has non-trivial normal subgroup.

If $n_3=4$ and $n_2=3$ in G, then this gives contradiction of number of element in G
So, (b) is correct.

Option (c) incorrect.
There doesn't exist $1-1$ group homomorphism from $\mathbb{Z}_{24}$ to S_8

as $\frac{\mathbb{Z}_{24}}{\{0\}}\approx\mathbb{Z}_{24}$ implies S_8 has cyclic subgroup of order 24 which is not possible.

Option (d) correct.

$n_3=1\ or\ 4$

Since G has subgroup of order 2 (Say H)

Case I:- If $n_3=1$ then G has normal subgroup of order 3
(say T)
Hence HT is subgroup of order 6.

Case II:- If $n_3=4$

Define $|clT|=4$

($\because\ \exists$ only 4-3 S.S.G $\because\ clcT$)

$\left\{H\le G|\ \exists g\in G \text{ such that } gH\,g^{-1}=T\right\}$

$$\Rightarrow \quad |N(T)|=\frac{O(G)}{|cl(T)|}=6$$

from case (i) and (ii)
We have subgroup of order 6
Correcrt option (b), (d)

Qus:- (ID 538)

Sol:- (2,3) $A' = \{0\}$ & $0 \notin A$, so A is not closed

$B' = \{0\}$ & $0 \in B$ so B is closed set

A is homeomorphic to z as both are countably infinite with derived set as empty set.

But B is not homeomorphic to z as $z' = \phi$ but $B' \neq \phi$

Qus:- (ID 527)
Sol:- (3)

(1) $f(z) = \dfrac{z}{1+|z|}$

$\Rightarrow \quad f(re^{i\theta}) = \dfrac{r(\cos\theta + i\sin\theta)}{1+r}$

$\Rightarrow \quad u(r,\theta) = \dfrac{r\cos\theta}{1+r}$ & $v(r,\theta) = \dfrac{r\sin\theta}{1+r}$

So, $u_r = \dfrac{[(1+r)-r]\cos\theta}{(1+r)^2} = \dfrac{\cos\theta}{(1+r)^2}$

$v_\theta = \dfrac{r\cos\theta}{(1+r)}$

$\Rightarrow \quad r u_r \neq v_\theta$ so Cauchy Riemann equation is not satisfied.

(2) $f(z) = (\cos\alpha)x - (\sin\alpha)y + i(\sin\alpha)x + (\cos\alpha)y$

$\Rightarrow \quad u(x,y) = (\cos\alpha)x - (\sin\alpha)y$

& $v(x,y) = (\sin\alpha)x + (\cos\alpha)y$

Hence $u_x = v_y = \cos\alpha$

$-u_y = v_x = \sin\alpha$

So Cauchy Riemann equation is satisfied. But in original question $u = \cos\alpha x - \sin\alpha y$ & $v = \sin\alpha x + \cos\alpha y$ so it is not correct.

(3) $f(z) = e^{-\frac{1}{z^4}}$

$\Rightarrow \quad f(re^{i\theta}) = e^{-\frac{1}{r^4 e^{i4\theta}}}$

$= e^{-\frac{e^{-i4\theta}}{r^4}}$

$= e^{-\left(\frac{\cos 4\theta - i\sin 4\theta}{r^4}\right)}$

$= e^{-\frac{\cos 4\theta}{r^4}} \cdot e^{i\frac{\sin 4\theta}{r^4}}$

$= e^{-\frac{\cos 4\theta}{r^4}}\left[\cos\left(\dfrac{\sin 4\theta}{r^4}\right) + i\sin\left(\dfrac{\sin 4\theta}{r^4}\right)\right]$

$\Rightarrow \quad u(r,\theta) = e^{-\left(\frac{\cos 4\theta}{r^4}\right)}\left[\cos\left(\dfrac{\sin 4\theta}{r^4}\right)\right]$ &

$v(r,\theta) = e^{-\frac{\cos 4\theta}{r^4}}\left[\sin\left(\dfrac{\sin 4\theta}{r^4}\right)\right]$

$$r u_r = r\left[\begin{array}{l} e^{-\frac{\cos 4\theta}{r^4}}\left(\dfrac{4\cos 4\theta}{r^5}\right)\cos\left(\dfrac{\sin 4\theta}{r^4}\right) \\ + e^{-\frac{\cos 4\theta}{r^4}}\left(-\sin\left(\dfrac{\sin 4\theta}{r^4}\right)\right)\left(\dfrac{-4\sin 4\theta}{r^5}\right) \end{array}\right]$$

$= v_\theta$

&

$$r v_r = r e^{-\frac{\cos 4\theta}{r^4}}\left[\begin{array}{l} \left(\dfrac{4\cos\theta}{r^5}\right)\left(\sin\left(\dfrac{\sin 4\theta}{r^4}\right)\right) + \\ \left(\cos\left(\dfrac{\sin 4\theta}{r^4}\right)\right)\left(\dfrac{-4\sin 4\theta}{r^5}\right) \end{array}\right]$$

$= -u_\theta$

So Cauchy Riemann equation is satisfied.

(4) $f(z) = x^2 + i y^2$

$\Rightarrow \quad u(x,y) = x^2$

& $v(x,y) = y^2$

$u_x = 2x$ & $v_y = 2y \Rightarrow u_x \neq v_y$

$\forall\, x, y \in R$ so Cauchy Riemann equation is not satisfied.

Qus:- (ID 530)

Sol:- (2,3) $f(z)$ is entire function such that $|z f(z) - 1 + e^z| \le 1 + |z|$ then as

$|1+z| \le 1+|z| \quad \forall z \in c$

$\Rightarrow \quad z f(z) - 1 + e^z = a + bz;\ |a| \le 1 \ \&\ |b| \le 1$

Now $z f(z) - 1 + e^z = a + bz$ (1)

By differentiating (1) successively we get

$f(z) + z f'(z) + e^z = b$ (2)

$\Rightarrow \quad 2f'(z) + z f''(z) + e^z = 0$ (3)

$\Rightarrow \quad 3f''(z) + z f'''(z) + e^z = 0$ (4)

By putting $z = 0$ in (3) & (4) we get

$2f'(0) + 1 = 0 \quad \Rightarrow f'(0) = -1/2$

& $3f''(0) + 1 = 0 \quad \Rightarrow f''(0) = -1/3$

Qus:- (ID 531)

Sol:- (2,4) Order of primitive root of 17 will be $\phi(17) = 17 - 1 = 16$

Order of $2 = 8 \ \because 2^8 \bmod 17 = 1$ so 2 is not primitive root of 17

If order $(a) = \bmod 17 = 16$ then

order $(a^2) \bmod 17 = \dfrac{16}{G.C.D(2,16)}$

$= \dfrac{16}{2} = 8$

So, if a is primitive root modulo 17 then a^2 is never a primitive root modulo 17.

Qus:- (ID 533)

Sol:- (4)

Qus:- (ID 532)

Sol:- (1,3)

Given $p > 3$ (prime)

and $N = p(p+2)(p+4)$

So , option (a) is correct

Take $p = 5,\ N = 5(5+2)(5+4)$

$5 \times 7 \times 3 \times 3$

$\Omega(N) = 4$

similarly for any prime $p > 3$ we conclude that

$\Omega(N) \ge 3$, (using fundamental theorem of Artihmetic)

So ,option (b) is incorrect

There doesn't exist any prime $p > 3$ such that $\Omega(N) = 3$ using (*)

So, option (c) is correct

So, option (d) is incorrect

Qus:- (ID 537)

Sol:- (3) If $A = \{(x,y) \in R^2 \mid x^2 + y^2 < 1\}$

$\cup \{(x,y) \in R^2 \mid (x-2)^2 + y^2 < 1\}$

then A is not connected but $\overline{A}$ is connected.

$A = \{(x,y) \in R^2 \mid x^2 + y^2 = 1\}$ and

$B = \{(x,y) \in R^2 \mid y = x\}$ are connected but

$A \cap B = \left\{\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right), \left(\dfrac{-1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}\right)\right\}$ is disconnected.

Union of two compact sets are always compact.

There are infinite continuous functions from Q^2 to $\{(0,0),(1,1)\}$

Qus:- (ID 536)

Sol:- (3)

If $f(x)$ satisfies the Eisenstein irreducible criterian, then $f(x)$ is irreducible, if not then it need not be irreducible so, $f(x)$ can sometimes be irreducible and can some times be reducible for rest option we have counter example by taking

$= f(x) = 2x^2 + 1 = 1 + 0.x + 2x^2$

So, correct option is (c)

Qus:- (ID 529)

Sol:- (1,2)

$$f(z) = \frac{(\sin z)^m}{(1-\cos z)^n} = \frac{\left(z - \dfrac{z^3}{3!} + \ldots\right)^m}{\left(1 - \left(1 - \dfrac{z^2}{2!} + \ldots\right)\right)^n}$$

$$= \frac{z^m\left(1-\frac{z^2}{3!}+...\right)^m}{z^{2n}\left(\frac{1}{2!}-\frac{z^2}{4!}+...\right)^n}$$

$$\Rightarrow \quad f(z)=\frac{z^{m-2n}\left(1-\frac{z^2}{3!}+....\right)^m}{\left(\frac{1}{2!}-\frac{z^2}{4!}+...\right)^n}$$

$$= \frac{\left(1-\frac{z^2}{3!}+...\right)^m}{z^{2n-m}\left(\frac{1}{2!}-\frac{z^2}{4!}...\right)^n}$$

so, if $2n-m\le 0 \Rightarrow m\ge 2n$ then at $z=0$ $f(z)$ has removable singularity.

Also if $2n-m>0$

$\Rightarrow$ $2n>m$ or $m<2n$ then $f(z)$ has pole of order $2n-m$

Qus:- (ID 543)
Sol:- (2,4)

$x^2\frac{\partial^2 u}{\partial x\,\partial y}+3y^2u=0$ with $u(x,0)=e^{\frac{1}{x}}$

Clearly given PDE is linear

Now, let $u(x,t)=X(x)\ Y(y)$ (i)

$\Rightarrow \quad x^2 X' Y' + 3y^2 X Y = 0$

$\Rightarrow \quad x^2 X' Y' = -3y^2 X Y$

$\Rightarrow \quad x^2\frac{X'}{X} = -3y^2\frac{Y}{Y'} = K$

$\Rightarrow \quad x^2 = \frac{dX}{X\,dx} = K$ (let)

$\Rightarrow \quad \frac{dX}{X} = K\frac{dx}{x^2}$

$\Rightarrow \quad \log X = \frac{-K}{x} + \log C_1$

$\Rightarrow \quad \log\left(\frac{X}{C_1}\right) = \frac{-K}{x}$

$\Rightarrow \quad \frac{X}{C_1} = e^{\frac{-K}{x}}$

$\Rightarrow \quad \boxed{X = C_1 e^{\frac{-K}{x}}}$

Now, $-3y^2\frac{Y}{Y_1} = K \Rightarrow -3y^2 = \frac{K\,dY}{Y\,dy}$

$\Rightarrow \quad \frac{-3y^2}{3} + \log C_2 = K\log Y$

$\Rightarrow \quad -y^3 = K\left(\log\left(\frac{y}{C_2}\right)\right)$

$\Rightarrow \quad \frac{-y^3}{K} = \frac{Y}{C_2}$

$\Rightarrow \quad C_2 e^{\frac{-y^3}{K}} = Y$

Putting in (i) we get

$u(x,t) = C_1 e^{\frac{-K}{x}}.C_2 e^{\frac{-y^3}{K}}$

$\Rightarrow \quad u(x,t) = C_1.C_2 e^{\frac{-K}{x}}.e^{\frac{-y^3}{K}}$

Now, $u(x,0)=e^{\frac{1}{x}} \Rightarrow C_1C_2 = 1$ & $\boxed{K=-1}$

$\Rightarrow \quad u(x,t) = e^{\frac{1}{x}}.e^{\frac{y^3}{1}} = e^{\frac{1}{x}}.e^{y^3}$

$\Rightarrow \quad u(1,1) = e.\,e = e^2$

Qus:- (ID 542)
Sol:- (2)

Qus:- (ID 539)
Sol:- (1,3)

Qus:- (ID 547)
Sol:- (1,4)

Qus:- (ID 546)
Sol:- (1)

Qus:- (ID 540)
Sol:- (1,2,4)

Qus:- (ID 541)

Sol:- (3) $W \equiv 0$ in $(a,b) \Rightarrow x_1, x_2$ are linearly dependent . W can not change sign in (a,b) and $W(t_0)=0$ for some $t_o \in (a,b) \Rightarrow W \equiv 0$ in (a,b).

But $W(t_0)=1$ for some $t_o \in (a,b) \not\Rightarrow W \equiv 1$ in (a,b)

Qus:- (ID 544)
Sol:- (1,2)

Qus:- (ID 549)
Sol:- (2) By using Leibnitz rule of differatition, we get $\int_0^x u(t)\,dt = 1$ (1)

But at 0 $u(0)=0$,$1 \neq 0$ $\Rightarrow$ no solution.

Qus:- (ID 548)
Sol:- (1,2,3)

Qus:- (ID 550)
Sol:- (3)

Qus:- (ID 545)
Sol:- (2,3,4)

Qus:- (554)
Sol:- (3,4)

Qus:- (ID 559)
Sol:- (1,2,3)

Qus:- (ID 567)
Sol:- (3)

Qus:- (ID 564)
Sol:- (1)

Qus:- (ID 552)
Sol:- (3,4)

Qus:- (ID 566)
Sol:- (1,2)

Qus:- (ID 568)
Sol:- (2,4)

Qus:- (ID 563)
Sol:- (4)

Qus:- (ID 565)
Sol:- (1)

Qus:- (ID 558)
Sol:- (1,2,3,4)

Qus:- (ID 553)
Sol:- (1,2,3)

Qus:- (ID 555)
Sol:- (1,2,3,4)

Qus:- (ID 556)
Sol:- (1)

Qus:- (ID 551)
Sol:- (1,4)

Qus:- (ID 560)
Sol:- (1,4)

Qus:- (ID 557)
Sol:- (1,2,4)

Qus:- (ID 561)
Sol:- (3,4)

Qus:- (ID 562)
Sol:- (1)