IIT JAM 2019 QUESTION PAPER

Q.1-Q10. carry one mark each.

Q1. Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m}$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(a) Both $\{a_n\}$ and $\{b_n\}$ are divergent

(b) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent

(c) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent

(d) Both $\{a_n\}$ and $\{b_n\}$ are convergent

Q2. Let $T \in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$. defined by

$$V = \{X \in M_{n\times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of V is

(a) $pn - \text{rank}(T)$ (b) $mn - p \text{ rank}(T)$

(c) $p(m - \text{rank}(T))$ (d) $p(n - \text{rank}(T))$

Q3. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f : \mathbb{R}^3 \to \mathbb{R}$ by

$f(x, y, z) = g(x^2 + y^2 - 2z^2)$.

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(a) $4(x^2 + y^2 - 4z^2)g''(x^2 + y^2 - 2z^2)$

(b) $4(x^2 + y^2 + 4z^2)g''(x^2 + y^2 - 2z^2)$

(c) $4(x^2 + y^2 - 2z^2)g''(x^2 + y^2 - 2z^2)$

(d) $4(x^2 + y^2 + 4z^2)g''(x^2 + y^2 - 2z^2)$
$+8g'(x^2 + y^2 - 2z^2)$

Q4. Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n x^n$

(a) converges for all x with $|x| < 2$

(b) converges for all with $|x| > 2$

(c) does not converge for any x with $|x| > 2$

(d) does not converge for any x with $|x| < 2$

Q5. Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(a) $\mathbb{Q}_+ \subseteq S$ (b) $S \subseteq \mathbb{Q}_+$

(c) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \phi$ (d) $S \cap \mathbb{Q}_+ \neq \phi$

Q6. If $x^h y^k$ is an intergrating factor of the differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$, then the ordered pair (h, k) is equal to

(a) $(-2, -2)$ (b) $(-2, -1)$

(c) $(-1, -2)$ (d) $(-1, -1)$

Q7. If $y(x) = \lambda e^{2x} + e^{\beta x}, \beta \neq 2,$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying $\frac{dy}{dx}(0) = 5,$ then $y(0)$ is equal to

(a) 1 (b) 4
(c) 5 (d) 9

Q8. The equation of the tangent plane to the surface $x^2z+\sqrt{8-x^2-y^2}=6$ at the point $(2,0,1)$ is

(a) $2x+z=5$ (b) $3x+4z=10$
(c) $3x-z=10$ (d) $7x-4z=10$

Q9. The value of the integral

$\int_{y=0}^{1}\int_{x=0}^{1-y^2} y\sin(\pi(1-x)^2)dxdy$ is

(a) $\frac{1}{2\pi}$ (b) 2π
(c) $\frac{\pi}{2}$ (d) $\frac{2}{\pi}$

Q10. The area of the surface generated by rotating the curve $x=y^3, 0\le y\le 1$, about the y-axis, is

(a) $\frac{\pi}{27}10^{3/2}$ (b) $\frac{4\pi}{3}(10^{3/2}-1)$
(c) $\frac{\pi}{27}(10^{3/2}-1)$ (d) $\frac{4\pi}{3}10^{3/2}$

Q11.-Q.30 carry two marks each

Q11. Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H\cap K$ is
(a) 3 (b) 4
(c) 6 (d) 12

Q12. Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2}+i$, with $i=\sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then
(a) $P^4=4P^2+9I$ (b) $P^4=4P^2-9I$
(c) $P^4=2P^2-9I$ (d) $P^4=2P^2+9I$

Q13. The set $\left\{\frac{x}{1+x}:-1<x<1\right\}$, as a subset of $\mathbb{R}$, is
(a) connected and compact
(b) connected but not compact
(c) not connected but compact
(d) neither connected nor compact

Q14 The set $\left\{\frac{1}{m}+\frac{1}{n}:m,n\in\mathbb{N}\right\}\cup\{0\}$, as a subset of $\mathbb{R}$, is
(a) compact and open
(b) compact but not open
(c) not compact but open
(d) neither compact nor open

Q15. For $-1,<x<1$, the sum of the power series $1+\sum_{n=2}^{\infty}(-1)^{n-1}n^2x^{n-1}$ is

(a) $\frac{1-x}{(1+x)^3}$ (b) $\frac{1+x^2}{(1+x)^4}$
(c) $\frac{1-x}{(1+x)^2}$ (d) $\frac{1+x^2}{(1+x)^3}$

Q16. Let $f(x)=(\ln x)^2, x>0$. Then
(a) $\lim_{x\to\infty}\frac{f(x)}{x}$ does not exist
(b) $\lim_{x\to\infty}f'(x)=2$
(c) $\lim_{x\to\infty}(f(x+1)-f(x))=0$
(d) $\lim_{x\to\infty}(f(x+1)-f(x))=0$ does not exist

Q17. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $f'(x)>f(x)$ for all $x\in\mathbb{R}$, and $f(0)=1$. Then $f(1)$ lies in the interval
(a) $(0,e^{-1})$ (b) $(e^{-1},\sqrt{e})$
(c) $(\sqrt{e},e)$ (d) (e,∞)

Q18. For which one of the following values of k, the equation $2x^3+3x^2-12x-k=0$ has three distinct real roots?
(a) 16 (b) 20
(c) 26 (d) 31

Q19. Which one of the following series is divergent?

(a) $\sum_{n=1}^{\infty}\frac{1}{n}\sin^2\frac{1}{n}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n}\log n$
(c) $\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{1}{n}$ (d) $\sum_{n=1}^{\infty}\frac{1}{n^2}\tan\frac{1}{n}$

Q20. Let S be the family of orthogonal trajectories of the family of curves

$2x^2 + y^2 = k$, for $k \in \mathbb{R}$ and $k > 0$. If $C \in S$ and C passes through the point (1,2), then C also passes through

(a) $(4, -\sqrt{2})$ (b) $(2, -4)$

(c) $(2, 2\sqrt{2})$ (d) $(4, 2\sqrt{2})$

Q21. Let $x, x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(a) $e + 1$ (b) $2e + 3$

(c) $3e + 2$ (d) $3e + 1$

Q22. The function $f(x, y) = x^3 + 2xy + y^3$ has a saddle point at

(a) (0,0) (b) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

(c) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (d) $(-1, 1)$

Q23. The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

(a) $\frac{\pi}{2}(17^{3/2} - 1)$ (b) $\pi(17^{3/2} - 1)$

(c) $\frac{\pi}{6}(17^{3/2} - 1)$ (d) $\frac{\pi}{3}(17^{3/2} - 1)$

Q24. Let C be the circle $(x-1)^2 + y^2 = 1$, oriented counter clockwise. Then the value of the line integral $\oint_C -\frac{4}{3}xy^3 dx + x^4 dy$ is

(a) 6π (b) 8π

(c) 12π (d) 14π

Q25. Let $\vec{F}(x, y, z) = 2y\hat{i} + x^2\hat{j} + xy\hat{k}$ and let C be the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Then the value of $\left|\oint_C F \cdot d\vec{r}\right|$ is

(a) π (b) $\frac{3\pi}{2}$

(c) 2π (d) 3π

Q26. The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point $(1,1,2)$ passes through

(a) $(-1, -2, 4)$ (b) $(-1, 4, 4)$

(c) $(3, 4, 4)$ (d) $(-1, 4, 0)$

Q27. The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Q28. Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if

(a) $\sum_{n=1}^{\infty} a_n^2$ converges

(b) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges

(c) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges

(d) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

Q29. For $\beta \in \mathbb{R}$, define

$$f(x, y) = \begin{cases} \dfrac{x^2 |x|^\beta y}{x^4 + y^2} & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then, at (0,0) the function f is

(a) continuous for $\beta = 0$

(b) continuous for $\beta > 0$

(c) not differentiable for any β.

(d) continuous for $\beta < 0$

Q30. Let $\{a_n\}$ be a sequence of positive real numbers such that

$a_1 = 1, a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0$ for all $n \geq 1$.

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(a) $(1,2]$ (b) $(2,3]$

(c) $(3,4]$ (d) $(4,5]$

Q31. - Q40. carry two marks each.

Q31. Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$

II. There is NO surjective (onto) homomorphism from from $\mathbb{Z}_8$ to G

(a) I is true (b) I is false

(c) II is true (d) II is false

Q32. Let G be a nonabelian group, $y \in G$, and let the map f, g, h from G to itself be defined by

$f(x) = yxy^{-1}, g(x) = x^{-1}$ and $h = g \circ g$.

Then

(a) g and h are homomorphism and f is not a homomorphism

(b) h is a homomorphism and g is not a homomorphism

(c) f is a homorphism and g is not a homomorphism

(d) f, g and h are homomorphism

Q33. Let S and T be linear transformation from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then

(a) rank(T) $\geq$ nullity(S)

(b) rank(S) $\geq$ nullity(T)

(c) rank(T) $\leq$ nullity(S)

(d) rank(S) $\leq$ nullity(T)

Q34. Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let G be a differentiable scalar function. Then

(a) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$

(b) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$

(c) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} - \nabla g \cdot \vec{F}$

(d) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} + \nabla g \cdot \vec{F}$

Q35. Consider the intervals $S = (0,2]$ and $T = [1,3)$.

Let S° and T° be the set of interior points of S and T, respectively. Then the set of interior points of S\T is equal to

(a) $S \setminus T^\circ$ (b) S\T

(c) $S^\circ \setminus T^\circ$ (d) $S^\circ \setminus T$

Q36. Let $\{a_n\}$ be the sequence given by

$$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, n \geq 1.$$

Then which of the folllowing statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$

(a) Both the subsequences are convergent

(b) Only one of the subsequences is convergent

(c) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$

(d) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

Q37. Let

$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x|$ and $g(x) = x^2$ for $x \in \mathbb{R}$.

If $h(x) = f(g(x))$, then

(a) h is not differentiable at $x = 0$

(b) $h'(\sqrt{\pi}) = 0$

(c) $h''(x) = 0$ has a solution in $(-\pi, \pi)$

(d) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q38. Let $f : \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by

$$f(x) = (\sin x)^\pi - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

(a) f is an increasing function

(b) f is a decreasing function

(c) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$

(d) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

Q39. Let

$$f(x,y)=\begin{cases}\dfrac{|x|}{|x|+|y|}\sqrt{x^4+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Then at (0,0)

(a) f is continuous

(b) $\dfrac{\partial f}{\partial x}=0$ and $\dfrac{\partial f}{\partial y}$ does not exist

(c) $\dfrac{\partial f}{\partial x}$ does not exist and $\dfrac{\partial f}{\partial x}=0$

(d) $\dfrac{\partial f}{\partial x}=0$ and $\dfrac{\partial f}{\partial y}=0$

Q40. Let $\{a_n\}$ be the sequence of real numbers such that $a_1=1$ and $a_{n+1}=a_n+a_n^2$ for all $n\geq 1$. Then

(a) $a_4=a_1(1+a_1)(1+a_2)(1+a_3)$

(b) $\lim\limits_{n\to\infty}\dfrac{1}{a_n}=0$

(c) $\lim\limits_{n\to\infty}\dfrac{1}{a_n}=1$

(d) $\lim\limits_{n\to\infty}a_n=0$

Q41.-Q50. carry one mark each.

Q41. Let x be the 100-cycle (1 2 3 ...100) and let y be the transposition (49 50) in the permutation group S_{100}. Then the order of xy is __________

Q42. Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$W_1=\{(x_1,x_2,\ldots,x_{100}):x_i=0$ if i is divisible by 4$\}$.

$W_2=\{(x_1,x_2,\ldots,x_{100}):x_i=0$ if i is divisible by 5$\}$.

Then the dimension of $W_1\cap W_2$ is ________

Q43. Consider the following system of three linear equations in four unknowns x_1,x_2,x_3 and x_4

$$x_1+x_2+x_3+x_4=4,$$
$$x_1+2x_2+3x_3+4x_4=5,$$
$$x_1+3x_2+5x_3+kx_4=5.$$

If the system has no solutions, then $k=$__________

Q44. Let $\vec{F}(x,y)=-y\vec{i}+x\vec{j}$ and let C be the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$ oriented counter clockwise. Then the value of $\oint_C\vec{F}\cdot d\vec{r}$ (round off to 2 decimal places) is __________

Q45. The coefficient of $\left(x-\dfrac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x)=\begin{cases}\dfrac{4(1-\sin x)}{2x-\pi}, & x\neq\dfrac{\pi}{2}\\ 0, & x=\dfrac{\pi}{2}\end{cases}$$

about $x=\dfrac{\pi}{2}$, is __________

Q46. Let $f:[0,1]\to\mathbb{R}$ be given by

$$f(x)=\frac{\left(1+x^{\frac{1}{3}}\right)^3+\left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$ Then

$\max\{f(x):x\in[0,1]\}-\min\{f(x):x\in[0,1]\}$ is __________

Q47. If $g(x)=\displaystyle\int_{x(x-2)}^{4x-5}f(t)dt$, where $f(x)=\sqrt{1+3x^4}$ for $x\in\mathbb{R}$ then $g'(1)=$__________

Q48. Let

$$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x^2-y^2}, & x^2-y^2\neq 0\\ 0, & x^2-y^2=0.\end{cases}$$

Then the directional derivative of f at (0,0) in the direction of $\dfrac{4}{5}\hat{i}+\dfrac{3}{5}\hat{j}$ is __________

Q49. The value of the integral

$$\int_{-1}^{1}\int_{-1}^{1}|x+y|\,dxdy$$

(round off to 2 decimal places) is __________

Q50. The volume of the solid bounded by the surfaces

$x = 1 - y^2$ and $x = y^2 - 1$, and the planes $z = 0$ and $z = 2$ (rounded off to 2 decimal places) is ____________

Q51-Q60. carry two marks each.

Q51. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (rounded off to 2 decimal places) is__________

Q52. The greatest lower bound of the set $\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\}$, (round off to 2 decimal places) is ______________

Q53. Let $G = \{n \in \mathbb{N} : n \le 55, \gcd(n, 55) = 1\}$ be the group under multiplication modulo 55.
Let $x \in G$ such that $x^2 = 26$ and $x > 30$. Then x is equal to____________

Q54. The number of critical points of the function $f(x,y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$ is __________

Q55. The number of elements in the set $\{x \in S_3 : x^4 = e\}$, where e is the identity element of the permutation group S_3, is__________

Q56. If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}, y, z \in \mathbb{R},$ is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$

then $z - y$ is equal to_______

Q57. Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then the maximum value of det(M) + det(N) is __________

Q58. Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma$ =-4, then $\alpha + \beta + \gamma$ is equal to __________

Q59. Let $y(x) = xv(x)$ be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0.$$

If $v(0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to ____________

Q60. If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx}4y = 0, \; y(0) = 2, \frac{dy}{dx}(0) = 0.$$

Then $y(\ln\,2)$ is (rounded off to 2 decimal places) equal to ___________

ANAND INSTITUTE OF MATHEMATICS

IIT JAM 2019 SOLUTION

Q1. (D)

Sol.: For

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log m)^m}$$

$$u_n = \frac{n}{(\log n)^n} < \frac{n}{2^n}, \forall n \geq 10$$

so by sandwich or squeeze theorem .

So, by Leibnitz test $\{a_n\}$ is convergent

Also for $b_n = \sum_{m=2}^{n} \frac{1}{(\log m)^m}$

$u_n = \frac{1}{(\log n)^n} < \frac{1}{2^n}$ sor sufficiently large n, so by comparison test for positive term series, $\{b_n\}$ is convergent.

Q2. (D)

Sol.: $T \in M_{m\times n}(R)$ and

$V = \left\{X \in M_{n\times p}(R) : TX = 0\right\}$

So, dimension of $V =$

$np -$ Number of linearly independent restrictions.

$= np -$ Rank(T).p

(because T will be coefficient matrix)

So, $\dim(V) = p(n - Rank(T))$

Q3. (B)

Sol.: $f(x,y,z) = g(x^2 + y^2 - 2z^2)$

$\Rightarrow \quad \frac{\partial f}{\partial x} = 2xg'(x^2 + y^2 - 2z^2)$

$\Rightarrow \quad \frac{\partial^2 f}{\partial x^2} = 2g'(x^2 + y^2 - 2z^2)$

$+4x^2 g''(x^2 + y^2 - 2z)$

$\frac{\partial f}{\partial y} = 2yg'(x^2 + y^2 - 2z^2)$

$\Rightarrow \quad \frac{\partial^2 f}{\partial y^2} = 2g'(x^2 + y^2 - 2z^2)$

$+4y^2 g''(x^2 + y^2 - 2z^2)$

$\frac{\partial f}{\partial z} = -4zg'(x^2 + y^2 - 2z^2)$

$\Rightarrow \quad \frac{\partial^2 f}{\partial z^2} = -4g'(x^2 + y^2 - 2z^2)$

$+16z^2 g''(x^2 + y^2 - 2z^2)$

So, $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$= 4(x^2 + y^2 + 4z^2)g''(x^2 + y^2 - 2z^2)$

Q4. (A)

Sol.: $na_n < b_n < n^2 a_n$

$\Rightarrow \quad (na_n)^{1/n} < b_n^{1/n} < (n^2 a_n)^{1/n}$ ...(i)

Now $\lim_{n\to\infty}(na_n)^{1/n}$

$= \lim_{n\to\infty} a_n^{1/n} \qquad \left(\because \lim_{n\to\infty} n^{1/n} = 1\right)$

Also $\lim_{n\to\infty}(n^2)^{1/n}(a_n)^{1/n} = \lim_{n\to\infty} a_n^{1/n}$

$\left(\because \lim_{n\to\infty}(n^2)^{1/n} = 1\right)$

So, By sandwich theorey on ...(i)

$\lim_{n\to\infty} b_n^{1/n} = \lim_{n\to\infty} a_n^{1/n}$

So, radius of convergence of the power series $\sum b_n x^n$ is also 4.

So it converges for all x with $|x| < 2$.

Q5. (B)

Sol.: Set of limit points of the set

$\left\{\frac{n}{\sqrt{2}}+\frac{\sqrt{2}}{n} : n \in N\right\}$ is $S=\phi$ so, $S \subseteq Q_+$

Q6. (A)

Sol.: $x^h y^k$ will be integrating factor of

$y(1+xy)dx + x(1-xy)dy = 0$. If

$(x^h y^{k+1} + x^{h+1}y^{k+2})dx + (x^{h+1}y^k - x^{h+2}y^{k+1})dy$
$= 0$ is exact

Here $M = x^h y^{k+1} + x^{h+1}y^{k+2}$

and $N = x^{h+1}y^k - x^{h+2}y^{k+1}$

$$\frac{\partial M}{\partial y} = (k+1)x^h y^k + (k+2)x^{h+1}y^{k+1}$$

$$\& \ \frac{\partial N}{\partial x} = (h+1)x^h y^k - (h+2)x^{h+1}y^{k+1}$$

Now $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ if

$k+1 = h+1$ & $k+2 = -(h+2)$

$\Rightarrow$ $h = k = -2$

So, $(h,k) = (-2,-2)$

Q7. (C)

Sol.: $y(x) = \lambda e^{2x} + e^{\beta x}$

$\Rightarrow$ $\frac{dy}{dx} = 2\lambda e^{2x} + \beta e^{\beta x}$

$\frac{dy}{dx}(0) = 5 \Rightarrow 2\lambda + \beta = 5$ ...(i)

Also Auxiliary equation of D.E. is

$m^2 + m - 6 = 0 \Rightarrow (m+3)(m-2) = 0$

$\Rightarrow$ $m = -3, 2$ so general solution is $c_1 e^{2x} + c_2 e^{-3x}$

so $\beta = -3$ ...(ii)

From (i) & (ii) $\lambda = 4$

$\Rightarrow$ $y(x) = 4e^{2x} + e^{-3x}$

$\Rightarrow$ $y(0) = 4 + 1 = 5.$

Q8. (B)

Sol.: $f(x,y,z) = x^2 z + \sqrt{8 - x^2 - y^4}$

$$\Rightarrow \frac{\partial f}{\partial x} = 2xz + \frac{1(-2x)}{2\sqrt{8-x^2-y^4}}$$

$$\Rightarrow \frac{\partial f}{\partial x}(2,0,1) = 3$$

$$\& \ \frac{\partial f}{\partial y} = \frac{1(-4y^3)}{2\sqrt{8-x^2-y^4}}$$

$$\Rightarrow \frac{\partial f}{\partial y}(2,0,1) = 0$$

$$\& \ \frac{\partial f}{\partial z} = x^2 \Rightarrow \frac{\partial f}{\partial z}(2,0,1) = 4$$

So the equation of the tangent plane is

$$(x-2)\frac{\partial f}{\partial x}(2,0,1) + (y-0)\frac{\partial f}{\partial y}(2,0,1)$$

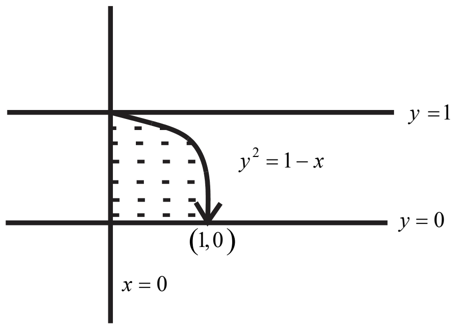
$$+(z-1)\frac{\partial f}{\partial z}(2,0,1) = 0$$

$\Rightarrow$ $(x-2)3 + (y-0).0 + (z-1)4 = 0$

$\Rightarrow$ $3x + 4z = 10$

Q9. (A)

Sol.:



$x = 1 - y^2 \Rightarrow y^2 = 1 - x$

$$I = \int_{y=0}^{1}\int_{x=0}^{1-y^2} y\sin(\pi(1-x)^2)dx\ dy$$

By changing order of Integration we get

$$I = \int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x}} y\sin(\pi(1-x)^2 dx\ dx$$

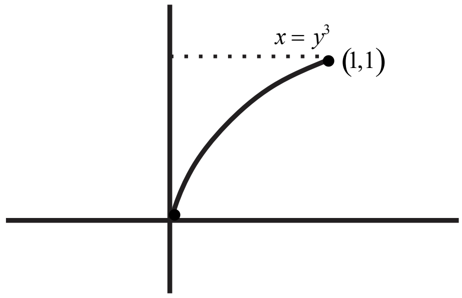
$$= \int_0^1 \frac{(1-x)}{2}\sin(\pi(1-x)^2)dx$$

$$= +\frac{1}{4\pi}\cos(\pi(1-x)^2)\ \Big|_0^1$$

$$= \frac{1}{4\pi}(1-(-1)) = \frac{1}{2\pi}$$

Q 10. (C)

Sol:



Area of the surface =

$$I = \int 2\pi x \, dS = \int 2\pi x\sqrt{(dx)^2 + (dy)^2}$$

$$= \int_{y=0}^{1} 2\pi y^3 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^1 2\pi y^3 \sqrt{1 + 9y^4}\, dy$$

Let $1 + 9y^4 = z \Rightarrow 36y^3 dy = dz$

At $y = 0$; $z = 1$ & $a + y = 1$; $z = 10$

$\Rightarrow$ $$I = \int_1^{10} \frac{\pi}{18}\sqrt{z}\, dz$$

$$= \frac{\pi}{18} \times \frac{z^{3/2}}{3/2}\Big|_1^{10} = \frac{\pi}{27} z^{3/2}\Big|_1^{10}$$

$$= \frac{\pi}{27}(10^{3/2} - 1).$$

Q11. (D)

Sol.: order of $H \cap K = G.C.D\,((0(H), 0(G))$

$= \text{G.C.D}(24, 36) = 12$

Q12. (C)

Sol.: One eigen values of rational matrix P is $\sqrt{2}$ +i so it"s other eigen values are $\sqrt{2} - i, -\sqrt{2} + i$ & $-\sqrt{2} - i,$ so it's characteristic equation is

$(x - \sqrt{2} - i)(x - \sqrt{2} + i)(x + \sqrt{2} - i)(x + \sqrt{2} + i)$ =0

$\Rightarrow$ $\left[(x - \sqrt{2})^2 + 1\right]\left[(x + \sqrt{2})^2 + 1\right] = 0$

$\Rightarrow$ $\left[x^2 + 3 - 2\sqrt{2}x\right]\left[x^2 + 3 + 2\sqrt{2}x\right] = 0$

$\Rightarrow$ $(x^2 + 3)^2 - 8x^2 = 0$

$\Rightarrow$ $x^4 - 2x^2 + 9 = 0$

So, By Cayley Hamilton theorem

$P^4 - 2P^2 - 9I = O$

$\Rightarrow$ $P^4 = 2P^2 - 9I.$

Q13. (B)

Sol.: $y = \dfrac{x}{1+x} = \dfrac{x + 1 - 1}{1 + x} = 1 - \dfrac{1}{1+x}$

Now $-1 < x < 1 \Rightarrow -\infty < y < \dfrac{1}{2}$

So, given set is $S = \left(-\infty, \dfrac{1}{2}\right)$

Here S is connected but it is not compact as it is neither closed nor bounded.

Q14. (B)

Sol.: $S = \left\{\dfrac{1}{m} + \dfrac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$

$\Rightarrow$ $S' = \left\{\dfrac{1}{p} \mid P \in \mathbb{N}\right\} \cup \{0\}$

As, $S' \subseteq S$, so S is closed set .Also, S is bounded, So S is compact set. But S is not open set.

Q15. (A)

Sol.: $S = 1 + \displaystyle\sum_{n=2}^{\infty} (-1)^{n-1} n^2 x^{n-1} = ?$

$\because$ $1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + \ldots = \dfrac{1}{1+x}$

So, differenting both sides with respect to x we get

$-1 + 2x - 3x^2 + \ldots + (-1)^n x^{n-1}.n = -\dfrac{1}{(1+x)^2}$

$\Rightarrow$ $x - 2x^2 + 3x^3 + \ldots + (-1)^{n-1}.nx^n = \dfrac{x}{(1+x)^2}$

Again differentiating w.r.t. x, we get

$1 - 2^2 x + 3^2 x^2 + \ldots + (-1)^{n-1}.n^2 x^{n-1} + \ldots$

$= \dfrac{1}{(1+x)^2} - \dfrac{2x}{(1+x)^3}$

$\Rightarrow$ $1 + \displaystyle\sum_{n=2}^{\infty} (-1)^{n-1}.n^2 x^{n-1} = \dfrac{(1-x)}{(1+x)^3}$

Q16. (C)

Sol.: $\lim\limits_{x\to\infty}(f(x+1) - f(x))$

$$= \lim_{x\to\infty} [\ln(x+1)]^2 - [\ln x]^2 \, [\infty - \infty \text{ case}]$$

$$= \lim_{x\to\infty} \left[\ln(x^2+x)\right]\left[\ln(1+\frac{1}{x})\right][\infty.0 \text{ case}]$$

$$= \lim_{x\to\infty} \frac{\ln(x^2+x)}{\dfrac{1}{\ln\left(1+\dfrac{1}{x}\right)}}\left[\frac{\infty}{\infty}\text{case}\right]$$

$$= \lim_{x\to\infty} \frac{2x+1}{(x^2+x)\left[-\dfrac{1}{\left(\ln\left(1+\dfrac{1}{x}\right)\right)^2}\times\dfrac{1}{\left(1+\dfrac{1}{x}\right)}\left(-\dfrac{1}{x^2}\right)\right]}$$

$$= \lim_{x\to\infty}(2x+1).\left(\ln\left(1+\frac{1}{x}\right)\right)^2$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\dfrac{1}{x}\right)^2}{\dfrac{1}{2x+1}}\left[\frac{0}{0}\text{case}\right]$$

$$= \lim_{x\to\infty} \frac{2\ln\left(1+\dfrac{1}{x}\right)\left[-\dfrac{1}{x^2}\right]}{-\dfrac{1}{(2x+1)^2}.2}$$

$$= \lim_{x\to\infty} 4\ln\left(1+\frac{1}{x}\right) = 0$$

Q17. (D)

Sol.: $f'(x) > f(x) \Rightarrow \dfrac{df(x)}{f(x)} > dx$

$\Rightarrow \quad \ln f(x) > x + c$

$\Rightarrow \quad f(x) > Ke^x$

Also $f(0) = 1$ so $f(x) > e^x$

$\Rightarrow \quad f(1) > e \Rightarrow f(1) \in (e, \infty)$.

Q18. (A)

Sol.: $f(x) = 2x^3 + 3x^2 - 12x - k$

$\Rightarrow \quad f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

sign scheme of $f'(x)$

$f(-\infty) = -\infty$ and $f(\infty) = \infty$

and $f(x)$ is strictly monotone in $(-\infty,-2),(-2,1)\,\&\,(1,\infty)$ so

$f(x) = 0$ will have 3 distinct real roots if $f(-2) > 0$ & $f(1) < 0$

Now $\quad f(-2) = -16 + 12 + 24 - k > 0$

$\Rightarrow \quad f(1) = 2 + 3 - 12 - k < 0 \Rightarrow k > -7$

So, $-7 < k < 20$

So among the given options $k = 16$

Q19. (B)

Sol.: $\because \dfrac{1}{n}\log n > \dfrac{1}{n}, \forall n \geq 3$

So by comparision test

$\sum_{n=1}^{\infty} \dfrac{1}{n}\log n$ is divergent

Q20. (C)

Sol.: Family of curves is

$2x^2 + y^2 = k$ whose differential equation is

$4x + 2y\dfrac{dy}{dx} = 0 \qquad \ldots$(i)

whose orthogonal trajectory has differential equation

$4x + 2y\left(-\dfrac{dx}{dy}\right) = 0$

$\Rightarrow \quad 2x = y\dfrac{dx}{dy} \Rightarrow 2\dfrac{dy}{y} = \dfrac{dx}{x}$

$\Rightarrow \quad 2\ln y = \ln x + \ln c$

$\Rightarrow \quad y^2 = cx$

It passes through (1,2) so

$\Rightarrow \quad (2)^2 = c.1 \Rightarrow c = 4$

$\Rightarrow \quad y^2 = 4x$

It also passes through point $\left(2, 2\sqrt{2}\right)$

Q21. (D)

Sol.: Let the differential equation be

$y''+ay'+by=Q(x)$ ...(i)

As $y=x$; $y=x+e^x$ & $y=1+x+e^x$ are solutions of (1) so

$a+bx=Q(x)$

$a+bx+e^x(1+a+b)=Q(x)$

from these equation

$b=0; a+b+1=0 \Rightarrow a=-1$

$\Rightarrow$ $Q(x)=-1$

So differential equation in(1) is

$y''-y'=-1$

whose auxiliary equation is

$m^2-m=0 \Rightarrow m=0,1$.

So, General solution is

$y(x)=c_1+c_2e^x+x$

$\Rightarrow$ $y'(x)=c_2e^x+1$

$y(0)=3 \Rightarrow c_1+c_2=3$

$y'(0)=4 \Rightarrow c_2+1=4$

$\Rightarrow$ $c_2=3$ & $c_1=0$

$\Rightarrow$ $y(x)=3e^x+x$

$\Rightarrow$ $y(1)=3e+1$

Q22. (A)

Sol.: $f(x,y)=x^3+2x+y^3$

$\Rightarrow$ $f_x=3x^2+2y$ & $f_y=2x+3y^2$

$f_x=0$ & $f_y=0 \Rightarrow (x,y)=(0,0)$ & $\left(-\frac{2}{3},-\frac{2}{3}\right)$

$f_{xx}=6x$ & $f_{yy}=6y$; $f_{xy}=2$

At $(0,0)$; $f_{xx}f_{yy}-f_{xy}^2$

$=36xy-(2)^2=-4<0$

So, $(0,0)$ is saddle point

Q23. (C)

Sol.: Equation of paraboloid is

$x^2+y^2+z=8 \Rightarrow z=8-x^2-y^2$

$\Rightarrow$ $z_x=-2x$; $z_y=-2y$

Now $\sqrt{z_x^2+z_y^2+1}=8 \Rightarrow z=8-x^2-y^2$

So, required surface area

$$=\iint_{x^2+y^2\le 4}\sqrt{1+4(x^2+y^2)}\,dxdy$$

$$=\int_{\theta=0}^{2\pi}\int_{r=0}^{2}\sqrt{1+4r^2}.r\ dr\ d\theta$$

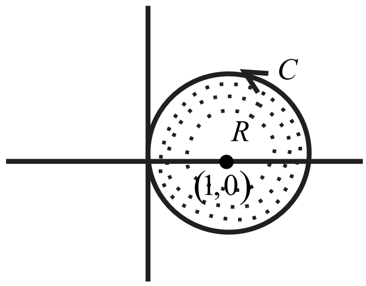
$$=\int_{\theta=0}^{2\pi}\frac{(1+4r^2)^{3/2}}{3/2}\times\frac{1}{8}d\theta$$

$$=(2\pi)\times\frac{1}{12}(4r^2+1)^{3/2}\Big|_0^2$$

$$=\frac{\pi}{6}\left(17^{3/2}-1\right)$$

Q24. (B)

Sol.:



$$P=-\frac{4}{3}xy^3 \Rightarrow \frac{\partial P}{\partial y}=-4xy^2$$

$$Q=x^4 \Rightarrow \frac{\partial Q}{\partial x}=4x^3$$

$$\Rightarrow \quad \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}=4x(x^2+y^2)$$

So, By Green's theorem in plane

$$\oint_C -\frac{4}{3}xy^3dx+x^4dy=$$

$$\iint_R 4x(x^2+y^2)dx\ dy$$

$$=\int_{\theta=0}^{2\pi}\int_{r=0}^{1}4(1+r\cos\theta)\left[(1+r\cos\theta)^2+(r\sin\theta)^2\right]rdrd\theta$$

$$=\int_{\theta=0}^{2\pi}\int_{r=0}^{1}4(1+r\cos\theta)\left[(1+r^2+2r\cos\theta)\right]r\ dr\ d\theta$$

$$=\int_0^{2\pi}4\left[\frac{1}{2}+\frac{1}{4}+\frac{2}{3}\cos\theta+\frac{1}{3}\cos\theta\right.$$

$$+\frac{1}{5}\cos\theta+\frac{1}{2}\cos^2\theta\Big]$$

$$=4\left(\frac{1}{2}+\frac{1}{4}\right)2\pi+2.4.\frac{1}{2}.\frac{\pi}{2}$$

$$=6\pi+2\pi=8\pi$$

Q25. (C)

Sol.: Along the curve of intersection of the plane $x+y+z=1$ and cylinder $x^2+y^2=1$, we have

$x=\cos\theta, y=\sin\theta, z=1-\cos\theta-\sin\theta$

$0\le\theta<2\pi$

$\Rightarrow$ $dx=-\sin\theta\ d\theta$; $dy=\cos\theta\ d\theta$

& $dz=(\sin\theta-\cos\theta)d\theta$.

$\vec{F}\cdot d\vec{r}=2ydx+x^2dy+xy\ dz$

$=\left[-2\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta(\sin\theta-\cos\theta\right]d\theta$

$\oint_C \vec{F}\cdot d\vec{r}=\int_0^{2\pi}-2\sin^2\theta+\cos^3\theta+\sin^2\theta$

$\cos\theta-\sin\theta\cos^2\theta d\ \theta$

$=\int_0^{2\pi}-2\sin^2\theta d\theta=-2.4.\frac{1}{2}.\frac{\pi}{2}=-2\pi$

$=\oint_C \vec{F}\cdot d\vec{r}=|-2\pi|=2\pi$

Q26. (B)

Sol.: For surface $f(x,y,z)=x^2+y^2-z$ normal vector is n_1= grad f = 2xi + 2y j - k

= 2i + 2 j - k at (1,1,2)

For surface $g(x,y,z)=x+z-3$ normal vector is n_2 = grad g = i + k = i + k at (1,1,2)

Now vector perpendicular to plane containing n_1 and n_2 is cross product of n_1 and n_2

= 2i - 3j -2k , So tangent line required is

$$\frac{(x-1)}{2}=\frac{(y-1)}{-3}=\frac{(z-2)}{-2}$$

as $(2)(2)+2(-3)+(-1)(-2)=0$

So this line passes through $(-1,4,4)$

Q27. (D)

Sol.: For $A=\begin{pmatrix}1&2\\4&3\end{pmatrix}$; trace(A) = 4 and det(A) = -5

but for $B=\begin{pmatrix}2&3\\1&4\end{pmatrix}$; trace(B) = 6

So, set of eigen values of A and B are different. For rest of the matrices trace and determinant are that of A

Q28. (C)

Sol.: If $\sum_{n=1}^{\infty}\frac{a_{n+1}}{a_n}$ converges then

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=0<1$$

So, by D'Alembert's ratio test

$\sum_{n=1}^{\infty}a_n$ is convergent

Q29. (B)

Sol.: $f(x,y)=\begin{cases}\dfrac{x^2|x|^\beta y}{x^4+y^2}; & x\ne0\\ 0; & x=0\end{cases}$

If $\beta\le0$ then

$\lim_{(x,y)\to(0,0)}f(x,y)$ does not exist

By taking the limit along family of curves $y=mx^2$. But if $\beta>0$ then

$\lim_{(x,y)\to(0,0)}f(x,y)$ exist and equals to 0.

Q30. (A)

Sol.: $a_1=1$ & $a_{n+1}^2-2a_na_{n+1}-a_n=0$

so, $a_{n+1}^2-2a_na_{n+1}-a_n=0$

$a_{n+1}^2-2a_na_{n+1}-a_n=0$

$\Rightarrow$ $a_{n+1}\left(a_{n+1}-2a_n\right)=a_n$

As, $\{a_n\}$ is possitive term sequece, so

$a_{n+1}-2a_n>0 \quad\Rightarrow\quad a_{n+1}>2a_n$

$\Rightarrow$ $a_2>2$; $a_3>2^2$; $a_4>2^3$,...... $a_n>2^{n-1}$

so, $\sum_{n=1}^{\infty}\frac{a_n}{3^n}>\sum_{n=1}^{\infty}\frac{2^{n-1}}{3^n}=\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

$$=\frac{1}{2}\times\frac{\frac{2}{3}}{1-\frac{2}{3}}=1$$

$= \sum_{hn=1}^{\infty} \frac{a_n}{3^n} > 1$

Also, $a_{n+1}^2 = 2a_n a_{n+1} + a_n$

$\Rightarrow \quad a_{n+1}^2 < 2a_n a_{n+1} + a_{n+1}$

$\Rightarrow \quad a_2 < 3, a_3 < 7, a_4 < 15, \ldots . a_n < 2^n - 1$

$\Rightarrow \quad \sum_{n=1}^{\infty} \frac{a_n}{3^n} < \sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n - \frac{1}{3^n}$

$= \frac{\frac{2}{3}}{1-\frac{2}{3}} - \frac{\frac{1}{3}}{1-\frac{1}{3}} = 2 - \frac{1}{2} = \frac{3}{2}$

$\Rightarrow \quad 1 < \sum_{n=1}^{\infty} \frac{a_n}{3^n} < \frac{3}{2}$

$\Rightarrow \quad \sum_{n=1}^{\infty} \frac{a_n}{3^n} \in (1, 2]$

Q31. (A,C)

Sol: As G is a non-cyclic group of order 4, so there can be no injective homomorphism from G to Z_8 and also there can be no surjective homomorphism from Z_8 to G.

Q32. (B,C)

Sol: $f(x) = y\,x\,y^{-1}$; $g(x) = x - 1$ and $h = go\,g$

$\Rightarrow \; h(x) = x$.

Here f and h are homomorphism from G to G but g is not homomorpshim.

Q33. (C,D)

Sol: S and T are linear transformations from V to V and $S(T(V)) = 0$.

If $\dim(V) = n$ then

Rank $(ST) \geq$ Rank $(S)+$ Rank $(T) - n$

$\Rightarrow \quad 0 \geq$ Rank $(S)+$ Rank $(T) - n$

$\Rightarrow$ nullity $(T) \geq$ Rank (S) &

nullity $(S) \geq$ Rank (T)

Q34. (A,D)

Sol: By the concept of box product $\nabla . \left(\vec{F} \times \vec{G}\right) = \vec{G} . \nabla \times \vec{F} - \vec{F} . \nabla \times \vec{G}$. Also by product rule $\nabla . \left(g\vec{F}\right) = g \nabla . \vec{F} + \nabla g . \vec{F}$.

Q35. (B,D)

Sol: $S = (0, 2]$ & $T = (1, 3]$

$\Rightarrow \quad S^\circ = (0, 2)$ & $T^\circ = (1, 3)$

Also, $S \setminus T = (0, 1)$

$\Rightarrow \quad (S \setminus T)^\circ = (0, 1)$

Also, $S^\circ \setminus T = (0, 1)$

$\Rightarrow \quad (S \setminus T)^\circ = S \setminus T = S^\circ \setminus T$.

Q36. (A)

Sol: $a_n = \max\left\{\sin\frac{n\pi}{3}, \cos\frac{n\pi}{3}\right\}$

$\Rightarrow \quad \{a_{6n-1}\} = \max\left\{-\sin\frac{\pi}{3}, \cos\frac{\pi}{3}\right\}$

$= \left\{\frac{1}{2}\right\}$

Which is constant sequence, so it is convergent and $\{a_{6n-1}\}$ converges to $\frac{1}{2}$

$\{a_{6n+4}\} = \max\left\{\sin\frac{4\pi}{3}, \cos\frac{4\pi}{3}\right\}$

$= \left\{-\frac{1}{2}\right\}$

Which is constant sequence, so it is convergent sequence and $\{a_{6n+4}\}$ converges to $-\frac{1}{2}$.

Q37. (B,C,D)

Sol: $f(x) = \cos(|\pi - x1|) + (x - \pi)\sin|x|$

$= -\cos x + (x - \pi)\sin x \; ; \; x \geq 0$

$= -\cos x + (\pi - x)\sin x \; ; \; x \leq 0$

$h(x) = f(g(x)) = f(x^2)$

$\Rightarrow \quad h(x) = -\cos x^2 + (x^2 - \pi)\sin x^2$

$\Rightarrow \quad h'(x) = x\sin x^2 + 2x\sin x^2 + 2x(x^2 - \pi)\cos x^2$

$\Rightarrow \quad h'(\sqrt{\pi}) = 0$ **(option B)**

Now, as $h : (-\pi, \pi) \to (-\pi, \pi)$ is differentiable function and hence continous function, so by fixed point theorem there exists $x_0 \in (-\pi, \pi)$

s.t. $h(x_0)=x_0$

(option D)

Also $h'(\sqrt{\pi})=h'(-\sqrt{\pi})=0$

So By Rolle's theorem $h''(x)$ has a solution in $(-\sqrt{\pi},\sqrt{\pi})$, hence it has a solution in $(-\pi,\pi)$.

(Option C)

Q38. (B,C)

Sol: $f(x)=(\sin x)^{\Pi}-\pi\sin x+\pi$

$\Rightarrow$ $f'(x)=\pi(\sin x)^{\pi-1}\cos x-\pi\cos x$

$=\pi\cos x\left((\sin x)^{\pi-1}-1\right)$

<0 in $\left(0,\dfrac{\pi}{2}\right)$

$\Rightarrow$ $f(x)$ is decreasing function in $\left(0,\dfrac{\pi}{2}\right)$

Also $f(0)=\pi$ & $f\left(\dfrac{\pi}{2}\right)=1$

so, $f(x)>0$ for all $x\in\left(0,\dfrac{\pi}{2}\right)$.

Q39. (A,D)

Sol: $f(x,y)=\begin{cases}\dfrac{|x|}{|x|+|y|}\sqrt{x^4+y^2}; & (x,y)\neq(0,0)\\ 0 \quad ; & (x,y)=(0,0)\end{cases}$

$\lim\limits_{(x,y)\to(0,0)} f(x,y)=$

$\lim\limits_{(x,y)\to 0^+}\dfrac{r|\cos|r^2\sqrt{r^2\cos^4\theta+\sin^2\theta}}{|\cos\theta|+|\sin\theta|}$

$=0$

So, At $(0,0)$ $f(x,y)$ is continuous.

$f_x(0,0)=\underset{h\to 0}{lt}\dfrac{|h|}{|h|}\sqrt{h^4}=0$

$f_y(0,0)=\underset{k\to 0}{lt}\dfrac{0}{|k|}\sqrt{k^2}=0$

Q40. (A,B)

Sol: $a_{n+1}=a_n+a_n^2=a_n(1+a_n)$

$\Rightarrow$ $a_2=a_1(1+a_1)$

$\Rightarrow$ $a_3=a_2(1+a_2)=a_1(1+a_1)(1+a_2)$

$a_4=a_3(1+a_3)=a_1(1+a_1)(1+a_2)(1+a_3)$

$\Rightarrow$ Sequence $\{a_n\}$ diverges to ∞, so $\lim\dfrac{1}{a_n}=0$.

Q41. (99)

Sol: $xy=(1,2,3,....100)(49\ 50)$ is from S_{100} then $(xy)^{100-1}=(xy)^{99}=I$ (identity).

So, order of xy is 99.

Q42. (60)

Sol: $W_1\cap W_2=\{x_1,x_2,....x_{100}\}\,|\,x_i=0$ if i is divisible by 4 or divisible by 5

Number of 0 tuples in $W_1\cap W_2$ will be

$\dfrac{100}{4}+\dfrac{100}{5}-\dfrac{100}{4.5}=25+20-5$

$=40$

So, number of independent tuples $=100-40=60$.

So, dimension of $W_1\cap W_2=60$.

Q43. (7)

Sol: The given system is

$\begin{bmatrix}1&1&1&1&4\\1&2&3&4&5\\1&3&5&K&5\end{bmatrix}\sim$

$\begin{bmatrix}1&1&1&1&4\\0&1&2&3&1\\0&2&4&K-1&1\end{bmatrix}\sim$

By $R_2\leftarrow R_2-R_1$ &

$R_3\leftarrow R_3-R_1$

$\begin{bmatrix}1&1&1&1&4\\0&1&2&3&1\\0&0&0&k-7&-1\end{bmatrix}$ by $R_3\leftarrow R_3-2R_2$

which will be inconsistent

If $k-7=0$ $\Rightarrow k=7$

Q44. (75.40)

Sol: $\vec{F}=-y\hat{i}+x\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

$\Rightarrow \quad d\vec{r} = dx\hat{i} + dy\hat{j}$

$\vec{F}.d\vec{r} = -y\,dx + x\,dy$

C is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\Rightarrow \quad I = \oint_C \vec{F}.d\vec{r} = \oint_C -y\,dx + x\,dy$

$= \iint_R 2\,dx\,dy$ (by Green's theorem)

Where R is region bounded by the ellipse.

$\Rightarrow \quad I = 2\pi(ab); \quad a = 4 \ \& \ b = 3$

$= 24\pi = 24(3.1416)$

$= 75.3984$

≈ 75.40

Q45. (1)

Sol:

$$f(x) = \begin{cases} \frac{4(1-\sin x)}{2x-\pi} & ; \ x \neq \frac{\pi}{2} \\ 0 & ; \ x = \frac{\pi}{2} \end{cases}$$

$$\Rightarrow \quad f'(x)|_{\frac{\pi}{2}} = f'\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}} \frac{f(x) - f\left(\frac{\pi}{2}\right)}{x - \frac{\pi}{2}}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{4(1-\sin x)}{2\left(x-\frac{\pi}{2}\right)^2} \left[\frac{0}{0} Case\right]$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-4\cos x}{4\left(x-\frac{\pi}{2}\right)} \left[\frac{0}{0} Case\right]$$

$$= \lim_{x\to\frac{\pi}{2}} \sin x = 1$$

So, coefficient of $\left(x - \frac{\pi}{2}\right)$ in Taylor series expansion of $f(x) = f'\left(\frac{\pi}{2}\right) = 1$

Q46. (0.25)

Sol: $f(x) = \dfrac{\left(1+x^{1/3}\right)^3 + \left(1-x^{1/3}\right)^3}{8(1+x)}$

$$= \frac{2\left(1+3x^{2/3}\right)}{8(1+x)} = \frac{1}{4}\left(\frac{1+3x^{2/3}}{1+x}\right)$$

As $f(x)$ is increasing function in $[0,1]$ so $f(0) = \frac{1}{4} = 0.25$ & $f(1) = \frac{1}{2} = 0.5$ are minimum and maximum in value of $f(x)$ in $[0,1]$.

So, $\quad \max\{f(x) : x \in [0,1]\}$

$\quad \min\{f(x) : x \in [0,1]\}$

$= 0.5 - 0.25 = 0.25$

Q47. (8)

Sol: $g(x) = \int_{x(x-2)}^{4x-5} f(t)\,dt$

$\Rightarrow \quad g'(x) = 4t(4x-5) - (2x-2)f(x(x-2))$

$= 4f(4x-5) - 2(x-1)f(x^2-2x)$

$\Rightarrow \quad g'(1) = 4f(-1) = 4\sqrt{1+3} = 8$

Q48. (2.6)

Sol:

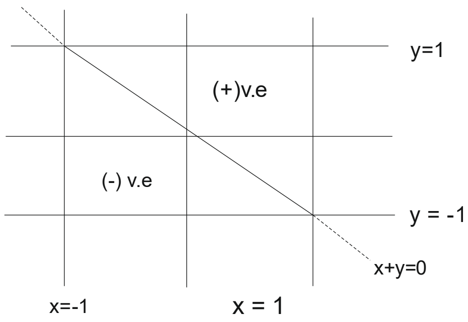
$$D_{\left(\frac{4}{3},\frac{3}{5}\right)}(0,0) = \lim_{t\to 0} \frac{f\left(\frac{4}{5}t, \frac{3}{5}t\right) - f(0,0)}{t}$$

$$= \lim_{t\to 0} \frac{\frac{\left(\frac{4^3+3^3}{5^3}\right).t^3}{\left(\frac{4^2-3^2}{5^2}\right).t^2} - 0}{t}$$

$$= \frac{\frac{91}{125}}{\frac{7}{25}} = \frac{13}{5} = 2.6$$

Q49. (2.67)

Sol: $I = \int_{-1}^{1}\int_{-1}^{1} |x+y|\,dx\,dy$



$$\Rightarrow \quad I = \int_{x=-1}^{1} \int_{y=-x}^{1} x + y \, dy \, dx$$

$$+ \int_{x=-1}^{1} \int_{y=-1}^{-x} -(x+y) \, dy \, dx$$

$$\Rightarrow \quad I = \int_{-1}^{1} x(1+x) + \frac{(1-x^2)}{2} dx$$

$$+I = \int_{-1}^{1} -x(1-x) + \frac{1}{2}(1-x^2) \, dx$$

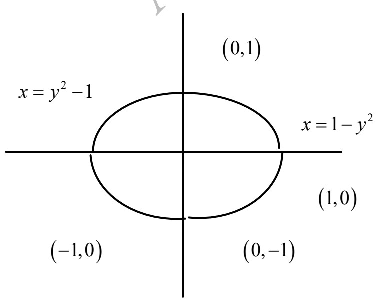
$$= \int_{-1}^{1} \frac{1+x^2}{2} dx + \int_{-1}^{1} \frac{(1+x^2)}{2} dx$$

$$= 2\int_{0}^{1} 1 + x^2 dx = 2\left(1 + \frac{1}{3}\right)$$

$$= \frac{8}{3} = 2.666, \ldots \simeq 2.67$$

Q50. (5.33)

Sol:



Required volume

$$= \int_{z=0}^{z=2} \int_{y=1}^{1} \int_{x=y^2-1}^{1-y^2} dx \, dy \, dz$$

$$= \int_{z=0}^{2} \int_{y=-1}^{1} 2(1-y^2) \, dy \, dx$$

$$= \int_{z=0}^{2} 4\left(1 - \frac{1}{3}\right) dz$$

$$= \frac{16}{3} = 5.33, \ldots$$

$$\approx 5.33$$

Q51. (6.7)

Sol: Required volume

$$= \int_{-2}^{0} \pi y^2 \, dx = \int_{-2}^{0} \pi x^4 (x+2) \, dx$$

$$= \pi \left[\frac{x^6}{6} + 2\frac{x^5}{5} \right]_{2}^{0}$$

$$= \pi \left[0 - \left(\frac{64}{6} - \frac{64}{5} \right) \right] = \frac{64\pi}{30}$$

$$= \frac{60(3.1416)}{30} = 64.(0.10427)$$

$$= 6.70208$$

$$\approx 6.7$$

Q52. (2.72)

Sol: Sequence $\left\{ \left(e^n + 2^n \right)^{1/n} \right\}$ is monotocially decreasing so it's greatest lower bound is.

$$\lim_{n\to\infty} \left(e^n + 2^n \right)^{1/n} =$$

$$\lim_{n\to\infty} \left[e^n \left(1 + \left(\frac{2}{n} \right)^n \right) \right]^{1/n} = e$$

$$\approx 2.71828 \ \approx 2.72$$

Q53. (31 or 46)

Sol: **As ,** $(31)^2 = 26(\mathrm{mod}\, 55)$

and $(46)^2 \equiv 26(\mathrm{mod}\, 55)$

So, $x = 31 \, or \, 46$

Q54. (5)

Sol: $f(x,y) = (x^2 + 3y^2) e^{-(x^2+y^2)}$

$$\Rightarrow \quad f_x = 2x e^{-(x^2+y^2)} - 2x(x^2+3y^2) e^{-(x^2+y^2)}$$

$$= 2x(1 - x^2 - 3y^2) e^{-(x^2+y^2)}$$

$$\& \quad f_y = 6y e^{-(x^2+y^2)} - 2y(x^2+3y^2) e^{-(x^2+y^2)}$$

$$= 2y e^{-(x^2+y^2)} (3 - x^2 - 3y^2)$$

$f_x = 0$ & $f_y = 0$ is simultaneously satisfied at critical point, so

$x = 0 \Rightarrow y = 0$ or $y = \pm 1$

$y = 0 \to x = 0$ or $x = \pm 1$

So, critical points are

$(0,0),(0,1),(0,-1),(1,0)$ & $(-1,0)$

So, there are 5 critical points.

Q55. **(4)**

Sol: $S = \{x \in S_3 \;;\; x^4 = e\}$

$S = \{(1,2),(1,3),(2,3),e\}$

So, number of elements in $S = 4$.

Q56. **(93)**

Sol: Let $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$ be eigenvector corresponding to eigen value λ then

$AX = \lambda X \Rightarrow$

$$\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ y \\ Z \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$$

$\Rightarrow \quad 2z = 2\lambda \quad \Rightarrow \quad \lambda = z$

$2 - 4z = \lambda y \quad \Rightarrow \quad 2 - 4z = yz$

& $y + 3z = \lambda z \quad \Rightarrow \quad y + 3z = z^2$

$\Rightarrow \quad yz + 3z^2 = z^3 \quad \Rightarrow \quad 2 - 4z + 3z^2 = z^3$

$\Rightarrow \quad z^3 - 3z^2 + 4z - z = 0$

$\Rightarrow \quad (z-1)(z^2 - 2z + 2) = 0$

$\Rightarrow \quad z = 1 \quad \Rightarrow \quad y = -2$

$\Rightarrow \quad z - y = 1 - (-2) = 3$

Q57. **(17)**

Sol:

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow \quad \det(MN) = 16$

$M \,\&\, N$ are Integral matrices, so $\det(M)$ & $\det(N)$ both are integers.

So, $\det(M) + \det(N)$ will be maximum if one of the value from $\det(M)$ & $\det(N)$ is 1 and other is 16, so maximum value of $\det(M) + \det(N)$ is $1 + 16 = 17$.

Q58. **(3)**

Sol $M^2 = M + 2I$

$\Rightarrow \quad M^2 - M - 2I = 0$

$\Rightarrow \quad x^2 - x - 2$ is an annihilating polynomial of M. As, $x^2 - x^2 - 2 = (x-2)(x+1) = 0$

$\Rightarrow \quad x = -1, 2$

If α, β, γ are eigenvalues of M, then their possible values are -1, & 2

So, $\alpha\beta\gamma = -4$

$\Rightarrow \quad \alpha = 2,\ \beta = 2,\ \gamma = -1$

So, $\alpha + \beta + \gamma = 2 + 2 - 1 = 3$

Q59. **(4)**

Sol: $y = xv$

$\Rightarrow \quad \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ **&** $\dfrac{d^2y}{dx^2} = x\dfrac{d^2v}{dx^2} + 2\dfrac{dy}{dx}$

Putting these in, $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = 0$

we get,

$$x^3\frac{d^2y}{dx^2} + 2x^2\frac{dv}{dx} - 3x^2\frac{dv}{dx} - 3xv + 3xv = 0$$

$\Rightarrow \quad x^3\dfrac{d^2y}{dx^2} = x^2\dfrac{dv}{dx}$

$\Rightarrow \quad x\dfrac{d^2v}{dx} = \dfrac{dv}{dx} \qquad$ (i)

Let $z = \dfrac{dv}{dx}$

$\Rightarrow \quad$ equation (i) becomes $x\dfrac{dz}{dx} = z$

$\Rightarrow \quad \dfrac{dz}{z} = \dfrac{dx}{x}$

$\Rightarrow \quad z = cx$

$\Rightarrow \quad \dfrac{dv}{dx} = cx$

$\Rightarrow \quad dv = cx\,dx$